On line reference trajectory adaptation for the control of a planar biped

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ABSTRACT

Assuming that a reference trajectory has been defined for a walking robot, a control law must be defined to follow this reference trajectory. But for a walking robot, in the presence of perturbation, the prevention of the fall down is of primary importance. This means that the forces exerted by the ground must be directed upwards to avoid unexpected take-off of a foot, the ratio between vertical and horizontal components must be such that sliding does not occur and the zero moment point must be on the support surface (or line). All these requirements are more important than a good convergence to the desired trajectory. In the studied strategy the tracking of a prescribed reference trajectory is replaced by the tracking of a trajectory belonging to a set of reference trajectories described by some parameters. And an on line adaptation of these parameters is allowed to preserve the equilibrium of the robot. When it is possible, without effect on the robot stability, the parameters go back to their desired value.

1. INTRODUCTION

Assuming that a reference trajectory has been defined for a walking robot, a control law must be defined to follow this reference trajectory. The computed torque control law has shown its efficiency for the control of robot manipulators and walking robots [2]. But for a walking robot, in the presence of perturbation, the prevention of the fall down is of primary importance [4]. This means that the forces exerted by the ground must be directed upwards to avoid unexpected take-off of a foot, the ratio between vertical and horizontal components must be such that sliding does not occur and the zero moment point must be on the support surface (or line). All these requirements are more important than a good convergence to the desired trajectory. The control of the Zero Moment Point [4] is a solution to prevent rotation of the supporting foot, some change of the trunk motion is accepted to satisfy a mechanical constraint.

In the studied strategy, this idea is generalised in two directions:

- a change of motions of all links is accepted, and this change is described by some parameters (one or two in the simulation, more parameters can be introduced in the methodology)
- the constraints corresponding to the avoidance of take-off, sliding, rotation of the foot are simultaneously taken into account. The limits of the actuators are also considered.

The methodology is the following. The tracking of a prescribed reference trajectory is replaced by the tracking of a trajectory belonging to a set of reference trajectories described by some parameters [3]. And an on line adaptation of these parameters is allowed to preserve the equilibrium of the robot. The parameters go back to their desired values if this does nit affect the mechanical stability of the robot.

In this paper, we first give the model of the studied biped and we express the mechanical constraints. In a third section, we present the considered set of reference trajectories. In the fourth section the control strategy is introduced. In section 5, some simulation results are given.

2. DESCRIPTION OF THE BIPED, GAIT AND MODELS

2.1. Modelling of the robot

The robot studied is a 7-link-planar robot shown in figure 1. The motion considered is composed of single support phases separated by impact phases. All links are assumed massive and rigid. The lengths of the thighs and of the shins are 0.4 m. However, their masses are different: 6 Kg for each thigh and 4 kg for each shin. The length of the torso is 0.625 m and its mass is 20 kg.

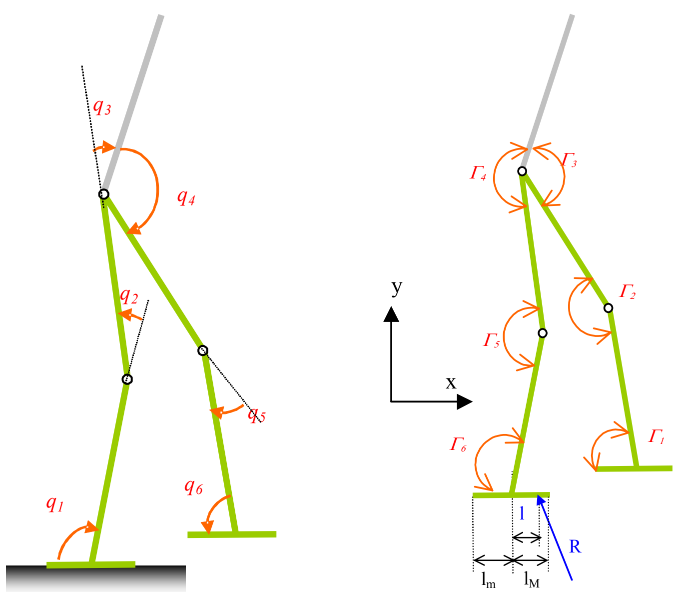

Fig. 1 The robot studied

In single support, the full sole of the supporting foot is in contact with the ground. The configuration of the robot can be described by 6 angular coordinates: $\mathbf{q} = [q_1,q_2,q_3,q_4,q_5,q_6]^t$. All these joints are actuated. Torques are applied on the haunches, knees and ankle. $\Gamma = [\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5, \Gamma_6]^t$.

In single support, the dynamic model can be written as a function of the **q** and its derivative only:

$$\mathbf{A}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{H}(\mathbf{q},\dot{\mathbf{q}}) = \mathbf{\Gamma} \tag{1}$$

where **A** is the inertia matrix, **H** is the vector of centrifuge, Coriolis and gravity effects.
The position of the centre of mass of the robot is a function of the joint co-ordinates **q**:

$$x_G = f_x(\mathbf{q}),\ y_G = f_y(\mathbf{q}) \tag{2}$$

2.2. The constraints

The global equilibrium of the robot defines de reaction force exerted by the ground $R=[R_x, R_y]^t$ on the supporting foot:

$$M\ddot{x}_G(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}}) = M\left(\frac{\partial f_x(\mathbf{q})}{\partial \mathbf{q}}\ddot{\mathbf{q}} + \frac{\partial^2 f_x(\mathbf{q})}{\partial \mathbf{q}^2}\dot{\mathbf{q}}^2\right) = R_x \tag{3}$$

$$M\ddot{y}_G(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}}) = M\left(\frac{\partial f_y(\mathbf{q})}{\partial \mathbf{q}}\ddot{\mathbf{q}} + \frac{\partial^2 f_y(\mathbf{q})}{\partial \mathbf{q}^2}\dot{\mathbf{q}}^2\right) = R_y - Mg \tag{4}$$

where M is the mass of the robot, g is the gravity acceleration.

But the ground can not apply any force on the robot, for example the ground does not glue the robot thus we have two conditions, one to avoid the take-off of the robot, the other one to avoid the sliding :

$$R_y>0, \qquad |R_x|<\mu\ R_y \tag{5}$$

where μ is the friction coefficient.

Using equations (3) and (4), these constraints equations are linear with respect to the joint acceleration and can be written:

$$\begin{gathered}
\left(-M\frac{\partial f_y(\mathbf{q})}{\partial \mathbf{q}}\right)\ddot{\mathbf{q}} + \left(-M\frac{\partial^2 f_y(\mathbf{q})}{\partial \mathbf{q}^2}\dot{\mathbf{q}}^2 - Mg\right) < 0 \\
\left(-\mu M\frac{\partial f_y(\mathbf{q})}{\partial \mathbf{q}} - M\frac{\partial f_x(\mathbf{q})}{\partial \mathbf{q}}\right)\ddot{\mathbf{q}} + \left(-\mu M\frac{\partial^2 f_y(\mathbf{q})}{\partial \mathbf{q}^2}\dot{\mathbf{q}}^2 - \mu Mg - M\frac{\partial^2 f_x(\mathbf{q})}{\partial \mathbf{q}^2}\dot{\mathbf{q}}^2\right) < 0 \\
\left(-\mu M\frac{\partial f_y(\mathbf{q})}{\partial \mathbf{q}} + M\frac{\partial f_x(\mathbf{q})}{\partial \mathbf{q}}\right)\ddot{\mathbf{q}} + \left(-\mu M\frac{\partial^2 f_y(\mathbf{q})}{\partial \mathbf{q}^2}\dot{\mathbf{q}}^2 - \mu Mg + M\frac{\partial^2 f_x(\mathbf{q})}{\partial \mathbf{q}^2}\dot{\mathbf{q}}^2\right) < 0
\end{gathered} \tag{6}$$

The foot in support must be motionless, thus the rotation equilibrium gives:

$$\Gamma_1 = lR_y \tag{7}$$

But the size is limited, thus the supporting ankle torque is limited to avoid a rotation of the foot:

$$l_m R_y < \Gamma_1 < l_M R_y \tag{8}$$

For the studied robot l_m=-0.1m, l_M=0.1m
The actuators have also some limits, thus we have for i=1 … 6

$$\Gamma_{im} < \Gamma_i < \Gamma_{iM} \tag{9}$$

The constraints (8), (9) are also linear with respect to the joint acceleration and can be rewritten using (1) and (4). All the 17 constraints presented can be written under the matrix form:

$$\mathbf{A_q}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{b_q}(\mathbf{q}, \dot{\mathbf{q}}) < 0_{17,1} \qquad (10)$$

where $\mathbf{A_q}$ is a (17x6) matrix, and $\mathbf{b_q}$ is a (17x1) vector.

3. THE SET OF REFERENCE TRAJECTORIES

3.1. Principle

Classically, a reference trajectory is known as a function of the time by the configuration $\mathbf{q^d}(t)$, velocity and acceleration of the robot at each time. In this paper, we consider a set of reference trajectories defined for different values of a parameter p (the stride for example) : $\mathbf{q^d}(p, t)$. These trajectories $\mathbf{q^d}(p, t)$ are assumed to be twice differentiable. The parameter p makes it possible to change the geometry of the reference trajectory. It is also possible to consider a time scaling of the trajectory, the description of the reference trajectory becomes: $\mathbf{q^d}(p, \tau(t))$, where $\tau(t)$ is a twice differentiable function of the time.
The evolution of the joint coordinates, of the velocity and acceleration for the considered set of reference trajectories can be written as function of parameters p and τ by:

$$\begin{cases} \mathbf{q^d} = \mathbf{q^d}(\mathrm{p}, \tau(\mathrm{t})), \\ \dot{\mathbf{q}}^\mathbf{d} = \dfrac{\partial \mathbf{q^d}}{\partial p}\dot{p} + \dfrac{\partial \mathbf{q^d}}{\partial \tau}\dot{\tau}, \\ \ddot{\mathbf{q}}^\mathbf{d} = \dfrac{\partial \mathbf{q^d}}{\partial p}\ddot{p} + \dfrac{\partial \mathbf{q^d}}{\partial \tau}\ddot{\tau} + \dfrac{\partial^2 \mathbf{q^d}}{\partial p^2}\dot{p}^2 + \dfrac{\partial^2 \mathbf{q^d}}{\partial \tau^2}\dot{\tau}^2 + 2\dfrac{\partial^2 \mathbf{q^d}}{\partial p \partial \tau}\dot{p}\dot{\tau}, \end{cases} \qquad (11)$$

In the following, $\ddot{p}$ and $\ddot{\tau}$ will be considered as new control inputs, and their evolution will enable us to insure mechanical stability (or respect of the constraints (20)) in the presence of disturbance.

3.2. An example

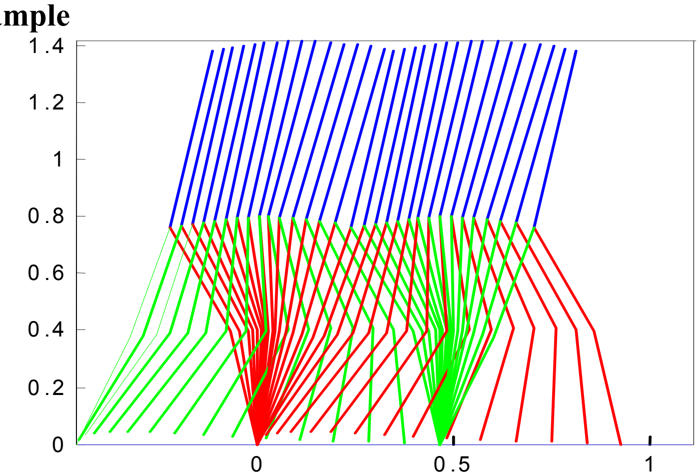

Fig. 2 The preferred trajectory

For a motion at 1m/s, using the technique proposed in [1], an optimal trajectory with respect to an energetic criteria has been defined: the stride is p^d=0.463 m, the duration of the step is T=0.463 s. The joint configurations are polynomials of time and the coefficients of the polynomials are the optimisation variables. For a single support phase, the result of the optimisation process is a joint evolution under the form

$$q_i(t)=a_{i0}+a_{i1}t+a_{i2}t^2+a_{i3}t^3+a_{i4}t^4=\sum_{j=0}^{4} a_{ij}t^j \quad \text{for } 0\leqslant t\leqslant T,\ i=1\ldots 6 \tag{12}$$

The gait includes an instantaneous impact with the ground, and the two legs exchange their role from one single support phase to the following one. In the optimisation process, the mechanical constraints are taken into account, thus without perturbations, this preferred motion can be followed.
The corresponding stick diagram is given in figure 2, the feet are not shown.

Imposing various values for p with $0.3\leqslant p\leqslant 0.65$ and for T=0.463 s, a family of sub-optimal reference trajectories has been derived using the same technique.
Corresponding to the different tested values of p, the value of the coefficients of a_{in} (i=1…6, n=0…4) are collected. Then, an interpolation, by polynomials functions of *p,* of the coefficients of the polynomials of time is done. As a result the reference joint evolution is:

$$q_i(t,p)=\sum_{j=0}^{4}\left(\sum_{k=0}^{7} b_{ijk}\,p^k\right)t^j \quad \text{for } 0\leqslant t\leqslant T,\ i=1\ldots 6 \tag{13}$$

The time scaling of this reference trajectory simply consists in replacing t by τ. Thus the set of reference trajectories is defined by

$$q_i(\tau(t),p(t))=\sum_{j=0}^{4}\left(\sum_{k=0}^{7} b_{ijk}\,p(t)^k\right)\tau(t)^j \quad \text{for } 0\leqslant t\leqslant T,\ i=1\ldots 6 \tag{14}$$

for any values of p and τ such that $0.3\leqslant p\leqslant 0.65$ and $0\leqslant \tau \leqslant 0.463$ s. The function: $\frac{\partial q^d}{\partial p},\frac{\partial q^d}{\partial \tau},\frac{\partial^2 q^d}{\partial p^2},\frac{\partial^2 q^d}{\partial \tau^2},\frac{\partial^2 q^d}{\partial p\partial \tau}$, can thus be easily deduced.

Remark: The trajectories defined by optimisation are such that the motions of the robot are cyclic, the duration of the single support phase is exactly T=0.463s, the impacts are taken into account. The introduction of the time scaling parameters will not change these characteristics [6] and the impact will occur when τ=0.463s. The interpolation used to define the joint variable as a continuous function of *p*, can introduce small errors on the continuity condition especially if $\dot{p}$ is different from zero at the impact time. In this case some tracking errors will appear after the impact, this tracking error will vanish quickly with the closed-loop control law.

4. THE STUDIED METHOD

The task of the walking robot is decomposed into primary and secondary tasks. Its first task is to follow a trajectory belonging to the previous set of reference trajectories and to preserve the contact with the ground during the single support phase. Its second task is to follow a preferred joint trajectory defined by a particular value $p=p^d$, and by $\dot{\tau}=1$.
At each time, the positions and velocities $\mathbf{q},\dot{\mathbf{q}},p,\dot{p},\tau,\dot{\tau}$ are measured or calculated, and the control inputs $\Gamma,\ddot{\tau},\ddot{p}$ are defined to achieve our objective.

4.1. The first task

A computed torque control law is used to follow a reference trajectory:

$$\mathbf{\Gamma} = \mathbf{A(q)w} + \mathbf{H(q,\dot{q})} \tag{15}$$

with

$$\mathbf{w} = \ddot{\mathbf{q}}^{\mathbf{d}} + \mathbf{K_v}\left(\dot{\mathbf{q}}^{\mathbf{d}} - \dot{\mathbf{q}}\right) + \mathbf{K_p}\left(\mathbf{q}^{\mathbf{d}} - \mathbf{q}\right) \tag{16}$$

where $\ddot{\mathbf{q}}^{\mathbf{d}}, \dot{\mathbf{q}}^{\mathbf{d}}, \mathbf{q}^{\mathbf{d}}$ are defined by equation (11).

The acceleration of the robot is:

$$\begin{aligned}
&\mathbf{w} = \frac{\partial \mathbf{q^d}}{\partial p}\ddot{p} + \frac{\partial \mathbf{q^d}}{\partial \tau}\ddot{\tau} + \mathbf{v}(\mathbf{q},\dot{\mathbf{q}}, p, \dot{p}, \tau, \dot{\tau}) \\
&\mathbf{v}(\mathbf{q},\dot{\mathbf{q}}, p, \dot{p}, \tau, \dot{\tau}) = \frac{\partial^2 \mathbf{q^d}}{\partial p^2}\dot{p}^2 + \frac{\partial^2 \mathbf{q^d}}{\partial \tau^2}\dot{\tau}^2 + 2\frac{\partial^2 \mathbf{q^d}}{\partial p \partial \tau}\dot{p}\dot{\tau} + \mathbf{K_v}\left(\dot{\mathbf{q}}^{\mathbf{d}} - \dot{\mathbf{q}}\right) + \mathbf{K_p}\left(\mathbf{q}^{\mathbf{d}} - \mathbf{q}\right)
\end{aligned} \tag{17}$$

where $\mathbf{v}(\mathbf{q},\dot{\mathbf{q}}, p, \dot{p}, \tau, \dot{\tau})$ is known.

Corresponding to a choice of $\ddot{p}, \ddot{\tau}$, the acceleration **w** and the torque Γ can be defined. This choice will also define the reaction force **R**.

This control law assumes that the contact with the ground exists but due to some perturbation, the contact can be interrupted (rotation of the foot, sliding or take-off). Thus the condition of contact must be checked. The conditions of contact are:

$$\mathbf{A_q(q)w} + \mathbf{b_q(q,q)} < 0_{17,1} \tag{18}$$

If one of these conditions is not satisfied, we propose to change **w** by changing $\ddot{p}, \ddot{\tau}$ to adapt the reference trajectory in order to preserve the contact with the ground.

4.2. The second task

If the current followed trajectory does not correspond to this preferred value, the trajectory will converge to the preferred trajectory by choosing:

$$\ddot{p} = \ddot{p}^*, \quad \ddot{\tau} = \ddot{\tau}^* \tag{19}$$

with

$$\ddot{p}^* = -k_v\dot{p} + k_p(p^d - p), \quad \ddot{\tau}^* = k_v(1 - \dot{\tau}) \tag{20}$$

And if the current followed trajectory is the preferred value, this choice insures the tracking of the preferred trajectory.

4.3. Combining first and second task

For the choice of $\ddot{p}, \ddot{\tau}$ corresponding to the second task (equations (19) and (20)), **w** is calculated. If **w** satisfies condition the constraints (18), the torques are calculated by (15) and are applied. If **w** does not satisfy condition (18), a new value of **w** is calculated to satisfy (18), this is the primary task, and to minimize $\|\ddot{p} - \ddot{p}^*\| + \|\ddot{\tau} - \ddot{\tau}^*\|$, which corresponds to the secondary task.

Since **w** is linear with respect to the parameters accelerations $\ddot{p}$ and $\ddot{\tau}$, and the constraints are linear with respect to **w** (see equation (18)). The constraints can be written:

$$\mathbf{A_p(q)}\begin{bmatrix}\ddot{p} \\ \ddot{\tau}\end{bmatrix} + \mathbf{b_p(q,q)} < 0_{17,1} \tag{21}$$

where $\mathbf{A_q}$ is a (17x2) matrix, and $\mathbf{b_q}$ is a (17x1) vector. The minimisation of a quadratic criterion under linear inequality constraints in a space of 1 or 2 dimensions can be done in a very fast way and thus can be implemented on-line.

If the constraints can not be satisfied, the on line modification is not able to cope with the tested perturbation and the robot will fall down.

The degrees of freedom added by the adaptation of the reference trajectory make it possible to cope with large perturbations that can not be accepted by a classical computed torque control law.

5. SIMULATION RESULTS

In simulation, without perturbation a perfect tracking of the preferred motion is followed.
A perturbation is considered, in which a force of 250N is applied in the back of the robot during 0.025s at the middle of the first step. During the application of an external force, a tracking error appears, the torques increase and the condition of no rotation of the foot is not naturally respected (see figure 3). The second derivative of the parameters p and τ are modified to preserve the contact (figures 4 and 5).

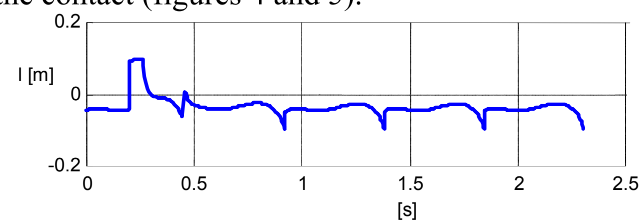

Fig. 3: Evolution of the position of the application point l

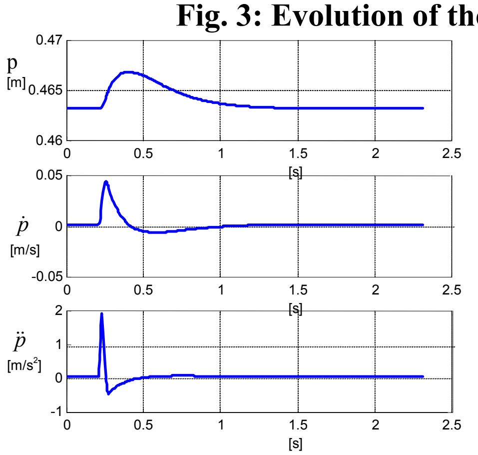

Fig 4: Evolution of parameter p

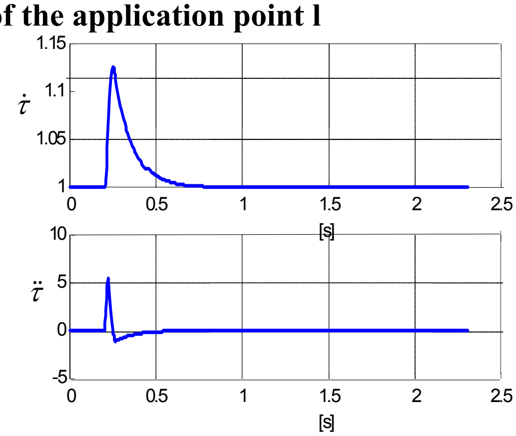

Fig 5: Evolution of parameter $\dot{\tau}$

During more than one step the real motion is not the preferred one but the robot is walking. Then the robot follows the preferred motion. The evolution of the tracking error for one variable is presented in figure 6. This error increases during the application of the external forces, then it decreases, but due to the variation of p and $\dot{p}$ after the impact some small errors exist. The evolution of one joint variable in its plan phase is presented in figure 7.

The robot is able to react without falling down for an external force (applied on the back during 0.025s) less than 650N when the two parameters τ and p are adapted. If only the parameter τ is adapted the maximal force is 455N, and if only the parameter p is adapted the maximal force is 320N,

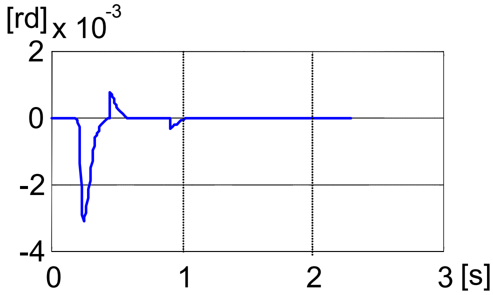

Fig 6: Tracking error for one variable

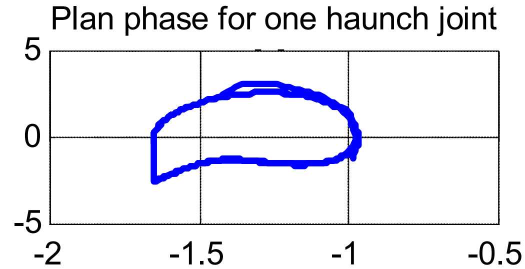

Fig 7: Evolution of one joint variable in its phase plane

6. CONCLUSION

An on line adaptation of the reference trajectory proposed without any test in [3] has been developed and tested on a seven-link planar robot. The adaptation can be done in various ways: by a change on the length of a step for example, by a change on the oscillations of the torso, a change on the duration of the step … In the presented simulations, two modifications are simultaneously taken into account: a change on the length of a stride, and a change of the duration of a step. These changes preserve the equilibrium of the robot and prevent its fall down in presence of perturbations.
In the present study, the length of the feet is 20 cm (centred on the ankle) for a leg of 80 cm. Future work will concern a robot without feet. A time scaling adaptation has already been conducted [6]. A geometric adaptation of the reference trajectory will be added to increase the robustness with respect to external perturbation. Some extends of this approach for the running will also be studied.

7. REFERENCES

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