Multi-Hypothesis Evasive Maneuvers for Safe Autonomous Navigation

Dimia Iberraken[1] and Lounis Adouane[2]

Abstract— **Recent advances in Autonomous Vehicles (AV) driving raised up all the importance to ensure the complete reliability of AV maneuvers even in highly dynamic and uncertain environments/situations. To reach this purpose, autonomous vehicles operating in such complex environments need methods which generalize to unpredictable situations. Validating the safety of self-driving vehicles can prove the coherence of the vehicles' behavior, reduce remaining risks and the need for extensive testing and, more importantly, allow us to plan evasive maneuver. This paper proposes a multi-hypothesis evasive strategy able to cope with any dynamic traffic situation. It is based on: a Sequential Decision Networks for Maneuver Selection and Verification (SDN-MSV) that calculates discrete evasive action maneuver based on defined situational criteria; an exhaustive evasive trajectory generation that considers multi-hypothesis kinematic and dynamic configuration; and on a multi-criteria optimization algorithm able to generate the corresponding low-level control that allows the ego-vehicle to pursue the advised collision-free evasive maneuver. At the same time, jerk is minimized while punishing high acceleration and curvature rate to provide enhanced comfort for passengers. Several simulation results show the good performance of the overall proposed evasive strategy.**

I. Introduction

A. Related works

Despite several years of developments of decision-making strategy for autonomous vehicles (AV) and the rich literature in this domain, there is unfortunately not yet a fully generic solution that deals with all kinds of scenarios. For this reason, recent advances in AVs raised all the importance of ensuring the complete safety [1], [2] of AV maneuvers even in highly dynamic and uncertain environments/situations. However, this objective becomes even more challenging due to the uniqueness of every traffic situation/condition. Indeed, the lack of safety guarantees proves, which is one of the key challenges to be addressed, limit drastically the ambition to introduce more broadly AVs in our roads, and restrict the use of AVs to very limited use cases. Extensive testing to simulate all possible behaviors of other traffic participants proves to be a time-consuming task. Indeed, considering the uniqueness of each traffic situation, the task of modeling every situation is nearly impossible. In addition, it can only prove that a system is unsafe, but is not able to propose an alternative. One solution in this case is to always verify the safety of the decided/planned maneuvers during the vehicle's navigation. This will give the ability to the system to abort automatically in case of any unexpected approaching objects, such as other objects and road users, entering the planned course of the vehicle. The vehicle must then be able to replan by determining an alternate route, i.e., the emergency trajectory, which the car will pursue instantly to avert an accident and guarantee safety all the time. This has been called in the literature online safety verification [3] or formal verification and answers to this challenge. It has been used in many works of the literature [3]–[6]. Because maneuvers are verified online while using safety verification techniques, the ability of the system to re-plan and evade a dangerous situation becomes possible. Emergency scenarios may necessitate maneuvering up to the vehicle's handling limits in order to avoid collisions [7]. The common used methods and the one from very early work related to emergency situations is to simultaneously plan a nominal and an emergency trajectory in order to guarantee the safety of the vehicle controller. With the help of this planning process, the vehicle controller is able to provide an emergency trajectory before and during the performance of a lane change or any other maneuver. Several literature researches tackle this problematic [8], [9]. However, generating an emergency maneuver for each time step is computationally expensive and often not needed, and an evasive strategy that is called as a last resort is preferable. Unlike other works, our approach proposes to plan evasive maneuver in real-time and guarantee safety with respect to any future motion of obstacles.

B. Contributions

Following up on our previous work [10] we present a multi-hypothesis evasive strategy to cope with any dynamic traffic situation. In the first place, the decision-making strategy is detailed in Section II and is based on a Bayesian Decision Network. It is designed to manage several road maneuvers under uncertainties. It utilizes defined risk metrics to propose discrete actions that allow to: derive appropriate maneuvers in a given traffic situation and provide a safety retrospection that updates in real-time the ego-vehicle movements according to the environment dynamics, in order to face any sudden hazardous and risky situation. In the latter case, it is proposed in Section III a multi-hypothesis evasive strategy able to cope with any dynamic traffic situation. It is based on: a Sequential Decision Networks for Maneuver Selection and Verification (SDN-MSV) that calculates discrete evasive action maneuver based on defined situational criteria, an exhaustive evasive trajectory generation that considers

This work was sponsored by a public grant overseen by the French National Research Agency as part of the "Investissements d'Avenir" through the IMobS3 Laboratory of Excellence (ANR-10-LABX-0016) and the IDEX-ISITE initiative CAP 20-25 (ANR-16-IDEX-0001).

[1] is with Université Clermont Auvergne, CNRS, SIGMA Clermont, Institut Pascal, F-63000 Clermont-Ferrand, France. dimia.iberraken@uca.fr

[2] is with Université de Technologie de Compiègne, CNRS, Heudiasyc, UMR 7253, 60203, Compiègne, France. lounis.adouane@hds.utc.fr

multi-hypothesis kinematic and dynamic configuration and a multi-criteria optimization algorithm able to generate the corresponding low-level control that allows the ego-vehicle to pursue the advised collision-free evasive maneuver. At the same time, we minimize jerk, punish high acceleration and curvature rate to provide enhanced comfort for passengers

II. Decision-Making Strategy

In this work, it is used a previously proposed probabilistic framework [11] that assesses the overall surrounding environment by evaluating the collision risk with all observed vehicles, plan driving maneuvers considering predictions of road user trajectories [12], make the decisions on the most suitable actions and investigate the possibility to decide discrete evasive actions if necessary [10]. The probabilistic decision-making framework is modeled as a sequencing of decisions that an autonomous vehicle should take by the means of a Sequential Decision Networks for Maneuver Selection and Verification (SDN-MSV). It is based on Bayesian Decision Network theory and has the ability to support probabilistic reasoning, decision-making under uncertainty for a given system and yield the capacity to incorporate multiple decision criteria [13]. In order to have a self-contained paper, it is given below an overview of the main elements constituting the overall decision-making strategy. The flowchart presented in Fig. 1 illustrates the different proposed decision/validation sequences and overall interactions between: the sequencing of decisions, an input risk assessment, an overall safety verification mechanism for all the obstacles present in the environment and the evasive strategy.

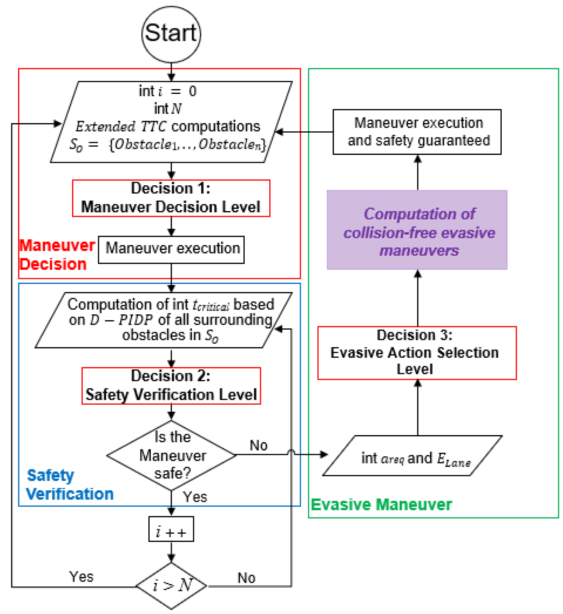

Fig. 1: Flowchart illustrating the sequencing of decisions and safety verification for all surrounding obstacles. The highlighted purple box contains the main components constituting the contributions of this paper. i is an integer value defining the iteration step. N is an integer value and is defined as $\left\lceil \frac{\mathbf{T_{ch}}}{\mathbf{T_s}} \right\rceil$ with T_s the sampling period and T_{ch} is the control horizon. S_O is the set of visible obstacles in the scene with memory tracking Id. $ETTC$ is the Extended Time To Collision. a_{req} is the required deceleration for emergency braking and E_{Lane} is the endangered lane.

Decision 1 - Maneuver Decision Level (MDL)

The first level decision (proposed in [11]) is a part of the Maneuver Decision Level (MDL) where at each time control horizon T_{ch} (set to 10 sampling time $10T_s$), the choice of action regarding the most suitable maneuver is made. The probabilistic decision process is based on the current risk assessment, using the ETTC (Extended Time to Collision, proposed in [11]) while taking measurement uncertainty into account. The ETTC gives an information of the collision risk at the time of the planning in 2D movement and not only when the vehicles' movements are collinear. The possible output maneuvers are: Lane Change Left (LCL), Lane Change Right (LCR), Keep Lane with ACC (KL-ACC), Maintain Velocity with CC (MV). These possible maneuvers are directly linked to the possible behavior that can be performed by dedicated controllers [11].

Decision 2 - Safety Verification Decision Level (SVDL)

The second level decision (proposed in [12]) is a part of the Safety Verification Decision Level (SVDL) where for each sampling time T_s, while the maneuver execution starts, a safety-checking regarding the action chosen in the MDL and a verification of the coherence of the maneuver with the predicted pre-planned trajectory is performed through the Dynamic Predicted Inter-Distance Profile (D-PIDP). The D-PIDP (Dynamic Predicted Inter-Distance Profile, proposed in [12]), is based on the study of the dynamic progression of the inter-distance between vehicles over a defined horizon. This decision level allows performing safety verification by quantifying the risks and the criticality of the driving situation beyond the remaining time to achieve the maneuver (defined as the horizon) in a retrospective manner. Its possible outputs are: Maneuver is Safe (MS) and Abort Maneuver (AM).

Decision 3 - Evasive Action Decision Level (EADL)

The third level decision is a part of the Evasive Action Decision Level (EADL) (proposed in [10]) where in case the verification procedure from the Safety Verification Decision Level (SVDL) advises to abort the maneuver, the system output the discrete evasive action based on two observations: the required deceleration a_{req} for emergency braking with regard to the vehicles' maximum capacities and on the endangered lanes E_{Lane} i.e., the lanes where the anomaly is detected. The possible outputs are: Emergency Braking (EB), Emergency Lane Change (ELC). This allows us to check if a braking maneuver alone is sufficient to avoid a collision, as this is often considered to be the most comfortable maneuver for passengers [2].

The overall network is updated as soon as new observations are available, and the most suitable decision is then obtained following the Expected utility theory that maximizes a utility function over the possible alternatives of the decision nodes given the available observation. More details on each layer Decision Bayesian network topology can be found in the authors' previous work [10]–[12]. The choice has been made in separating these levels of decision in the objective of being consistent i.e., while avoiding unnecessary switch in the plan. This allows the maneuver to be executed long enough before another decision is given when the situation is not changed significantly. In previous work [10], the decided action in the EADL output of the SDN-MSV has been applied to the system with a constant velocity configuration, and this while having an already defined fixed path to follow, which limits the flexibility of the evasion. In addition, the formalization of the constraints (related to the steering and the acceleration limitations of the ego-vehicle, the smoothness and comfort of the maneuvers) and the guarantee of safety has not been addressed.

In this work, the evasive strategy is further developed (cf. section III), and is proposed an exhaustive evasive trajectory generation that considers multi-hypothesis kinematic and dynamic configuration. Furthermore, a multi-criteria optimization is performed that takes into account the mentioned exhaustive process and is able to generate the corresponding low-level control that allows the ego-vehicle to pursue the safest and most comfortable advised collision-free evasive maneuver. Finally, a loop-back from the evasive maneuver is made towards the initialization of the algorithm once the safety state is reached to restart the decision-making process.

III. Evasive strategy

A. Problem statement

During the maneuver achievement, it is necessary to foresee possible refuge maneuvers, to deal with sudden detection of anomalies/threats, which can lead to risky situation. This is performed through the Sequential Decision Networks for Maneuver Selection and Verification (SDN-MSV) where the third decision level called Evasive Action Decision Level (EADL) is proposed in order to select the evasive maneuver/behavior which should be activated. The diagram given in Fig. 2 illustrates the procedure for computing these evasive maneuvers. We assume to be known a priori the initial state x, the set of surrounding Obstacle-vehicles S_O, the reference trajectory and the lanes' information as well as the trajectory predictions. We also assume in the diagram that the vehicle is at the right most lane to better exemplify our methodology. Note that motion planning and prediction is not the focus of this work; readers are referred to [11].

In the case the evasive decision is to perform an emergency braking, applying a_{req} on the ego-vehicle is sufficient to guarantee safety. This is feasible since the longitudinal constraints required in order to reach a desired stopping inter-distance are already satisfied by the procedure to deduce the required deceleration $a_{req} \leq a_{max}$ (the soundness of a_{req} has been shown in [10]). The lateral constraints are satisfied thanks to the already developed controller for lane keeping within the global architecture Probabilistic Multi-Controller Architecture (P-MCA) [11].

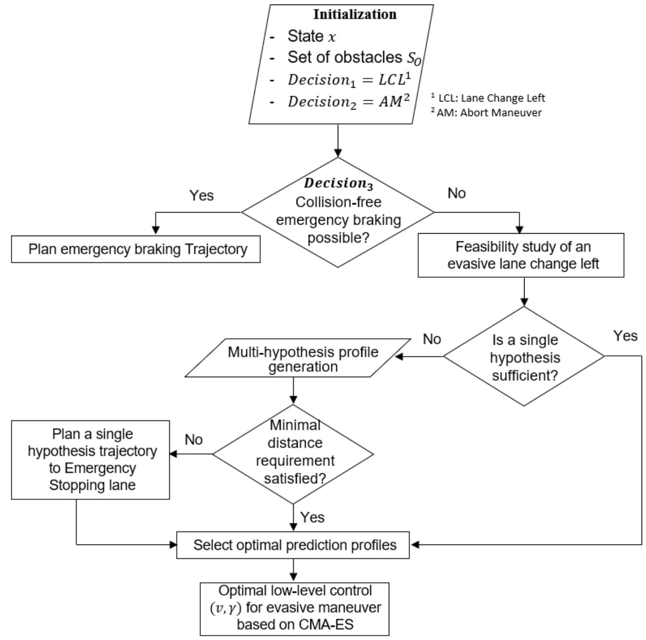

Fig. 2: Overall procedure for computing evasive maneuvers

Otherwise, a collision may be avoided by swerving to another lane. This swerve can be performed either by changing lane to the right or to the left. In this case study, the lane change right leads to the Emergency Stopping Lane (ESL). However, the proposed overall methodology could be applied if any other alternative exists, such as another free lane in the environment, or in general any other free space. For these situations, a single hypothesis (nominal trajectory) is generated to check at first if an evasive lane change to the left is sufficient to avoid collisions.

Otherwise, an exhaustive evasive lane change trajectory generation over a prediction horizon T_{pred} is performed with multiple-hypothesis kinematic and dynamic configuration in order to find the set of possible and feasible trajectories. Because an ESL exists explicitly in the environment in this case study and in case a lane change left is not feasible, a single hypothesis lane change right to the ESL is performed. To better understand the proposed methodology, in what follows it is presented the case of a single hypothesis (nominal trajectory) evasive prediction profile compared to the multiple-hypothesis configuration used in this paper in emergency situations.

B. Single hypothesis evasive prediction profile

After the EADL outputs the emergency lane change evasive decision, it is first checked if a lane change maneuver to the left is possible with the nominal configuration of the state and velocity at the time of the anomaly. The ego vehicle trajectory for lane change maneuvers is dimensioned using an obstacle avoidance method called Elliptic Limit Cycles [11] that is characterized by an elliptic orbit surrounding the obstacle. If well-dimensioned, the resulting trajectory generated around this orbit, guarantee if precisely followed the avoidance of any obstacle. The elliptic orbit is described by a major and a minor axis. These axes have bee dimensioned to take into account a longitudinal temporal safety distance t_s for the major axis, and a minimum lateral distance $L_{distance}$ for the minor axis. On the other hand, we suppose that the obstacle-vehicles follow a global path already defined to be the center-line of the lane. The adequate predicted trajectories (of the concerned vehicle pair) are generated over the prediction horizon T_{pred} based on the above definition while taking the nominal configuration of the velocity and initial state at the time of the anomaly. These predictions are used in order to define a reference predicted inter-distance profile (called Optimal Predicted Inter-Distance Profile (OPIDP)) and a reference predicted angular profile (called Optimal Predicted Angular Profile (OPAP)) to follow, that allow us (if precisely followed) to find the best control sequence $\mathbf{u(t)}$ (through the use of a multi-criteria optimization (cf. subsection III-D)) in order to perform the safe evasive action. These prediction profiles are calculated between the consecutive points of the predicted state vector of the ego-vehicle and the predicted state vector of the chosen obstacle-vehicle for each time step of the prediction as shown in Fig. 3 through $(p(t_0), p(t_1), p(t_2), p(t_3))$. The references are calculated for each endangered vehicle pair (ego/obstacle–vehicle). They are updated as soon as the used predictions are imprecise, or another anomalies is detected through the SVDL. Thus, as long as the proposed evasive strategy has enough good prediction of the movement of the obstacle-vehicle, the pertinence of OPIDP and OPAP are completely justified.

In summary, the optimal profiles OPIDP and OPAP allows, if precisely followed, to deduce two properties:

- **Property 1** Ensure a safe evasion since the future progress of the OPIDP must always ensure that the vehicle will never have inter-distance lower than the distance d_{min} (cf. Fig. 4). This distance can only be reached for example during a lane change maneuver when the ego-vehicle is in the adjacent lane and the vehicles are side by side. This latter property, is satisfied by the shape and dimensioning of the lane change trajectory.
- **Property 2** Constrain the vehicle to stay within the road range. The need to formalize an angular profile arised after the first test while using only the OPIDP. This test showed us that the vehicle is able to find a solution $u(t)$ that follows the defined OPIDP while respecting the defined set-points, but goes outside the road. In conclusion, as obvious as it may seem, the vehicle needs to respect not only the predicted inter-distance profile but also an appropriate angular profile to be able to stay within the road range.

An example of the resulting profiles is shown in the flowchart given in Fig. 4 for a given vehicle configuration. In this two lane configuration, the ahead obstacle-vehicle in front suddenly brakes and comes to standstill and the adjacent lane is free. In this case, the single hypothesis that we have put of a lane change left is feasible as the minimal distance thresholds d_{min} is guaranteed as we can see it in the figure. However, in another situation, where for example in addition to the anomaly of the ahead obstacle-vehicle, a fast vehicle is detected coming from behind in the left lane, the single hypothesis is not sufficient as will be seen in the simulation results (cf. IV-A).

C. Multi-hypothesis evasive prediction profile

For the case of emergency lane change, an exhaustive evasive lane change trajectory generation over a prediction horizon is performed with multiple-hypothesis kinematic and dynamic configuration (cf. subsection III-C.1 and III-C.2) in order to find the Set of Possible and Feasible Trajectories (Called S_{Ego}). The resulting set is used in order to compute the feasible set of predicted inter-distance profile. The predicted inter-distance is calculated between the set of generated trajectories of the ego-vehicle and the predicted trajectory of the targeted obstacle-vehicle following the same principle used in the single hypothesis use case for each individual trajectory from the set. An additional filtering stage is performed to remove the profiles that violates a minimal distance requirement d_{min} in order to keep the collision-free profiles. These profiles set are called Set of Predicted Inter-Distance Profile (S_{PIDP}). In what follows, is detailed the lateral and longitudinal parameters used to generate S_{Ego}.

1) Lateral motion parameters for trajectory generation: Lateral acceleration is caused by turning or by making lane changes. If the vehicle can maintain an appropriate speed when approaching the turn of a road, the lateral force will be limited, and passengers will feel more comfortable. As explained in section, the used lane change trajectory generation strategy is based on Elliptic Limit cycles (ELC) that have as parameters a longitudinal temporal safety distance t_s for the major axis, and a minimum lateral distance $L_{distance}$ for the minor axis. We considered in this work, the formalization of the minimal lateral distance proposed in the Responsibility Sensitive Safety (RSS) framework [1]. This

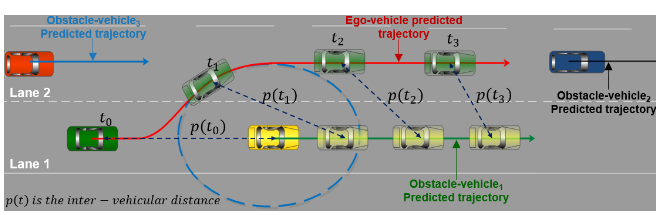

Fig. 3: Predicted Trajectories during lane change maneuver

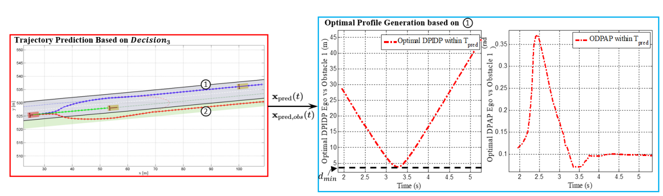

Fig. 4: Single Hypothesis Prediction Profile.

formalization takes into account the maximum and minim lateral acceleration possible, which is compatible with our reflection.

$$d_{\min}^{lat} = \mu + \left[\frac{v_1 + v_{1,\rho}}{2}\rho + \frac{v_{1,\rho}^2}{2a_{\text{min,brake}}^{\text{lat}}} - \left(\frac{v_2 + v_{2,\rho}}{2}\rho - \frac{v_{2,\rho}^2}{2a_{\text{min , brake}}^{\text{lat}}} \right) \right] \tag{1}$$

v_1 and v_2 be the lateral velocities of the vehicles c_1 and c_2. $v_{1,\rho}$ denotes $v_1 + \rho a^{\text{lat}}_{\text{max , acce } e}$ and $v_{2,\rho}$ denotes $v_2 - \rho a^{\text{lat}}_{\text{max accel}}$. The reaction time is given by ρ. The lateral safe distance d_min is then the distance required such that both vehicles can apply an acceleration with towards each other during the reaction time ρ, then minimally decelerate until zero lateral velocity, while still maintaining at least a μ distance. Taking the lateral acceleration applied in the literature as reference, the bounds of this study's (cf. Table I) have been fixed based on the baseline values provided by NHTSA's definition of a Near-Crash [14] which states that any circumstance that requires a rapid, evasive maneuver by the subject vehicle (or any other vehicle, pedestrian, cyclist, or animal) to avoid a crash is defined as steering, braking, accelerating, or any combination of control inputs that approaches the limits of the vehicle's capabilities. As a guide, subject vehicle braking greater than 0.5g or steering input that results in a lateral acceleration greater than 0.4g to avoid a crash constitutes a rapid maneuver.

2) Longitudinal motion parameters for trajectory generation: As for the longitudinal motion, the trajectory prediction were varied longitudinally in two ways. By varying the longitudinal acceleration and by varying the longitudinal temporal safety distance t_s of the defined ELC. Taking the longitudinal acceleration applied in the literature as reference [15], [16], the bounds of this study's (cf. Table I) have been fixed while taking into account the standard maximum/minimum comfortable acceleration/deceleration used in the literature. The authors in [16] performed safety-critical event extraction with regards to kinematic criteria from the Shanghai Naturalistic Driving Study (SH-NDS) and the calibration in our study have been inspired from these analyses. We chose $a_{min,accel}$ to represent accelerations at the limit of comfort, and $a_{max,accel}$ was chosen to represent a hard, uncomfortable accelerations. The same goes for $a_{min,brake}$ and $a_{max,brake}$. The prediction are also bounded to a maximal velocity v_{max}. The standard maximum comfortable deceleration is usually fixed between -3 to -3.5 m/s2. On the other side, the maximum deceleration value is obtained from the values of tire friction on dry condition for an automobile which is μ_{auto} = 0.8 which gives $a_{max} = -7.84 m/s^2$ by assuming $g = 9.8 m/s^2$. The bounds of the longitudinal temporal safety distance t_s was chosen based on the human driver minimum TTC that is distributed between 0.5 and 3 s in safety critical event according to the analysis performed on the SH-NDS data [16].

3) The choice of prediction profiles: The resulting overall set of trajectories S_{Ego} are shown in Fig. 5 and were generated while performing an iterative process over the various configuration detailed above. A filtering stage is performed to remove the profiles that violates the minimal distance requirement d_{min} in order to keep collision-free profiles.

An important challenge in the field of autonomous vehicle's risk management is to find the optimal balance between criteria like: smooth navigation, guaranteeing the comfort of the passenger or high level of safety insurance, and imposed constraints such as: to deal with uncertainties and complexity of the task to achieve, to respect the traffic laws and not being too conservative in the navigation. For this reason, we choose to select the optimal profile the closest to the middle of the feasible bounds. This is justified by the fact that we want to favor smooth and comfortable trajectories but also maintain a high margin of action to account for uncertainties and possible future changes in the environment. The resulting set of predicted inter-distance profile (S_{PIDP}) is shown in Fig. 6 and highlighted in purple the Optimal Predicted Inter-distance Profile (*OPIDP*).

The Optimal Predicted Angular Profile (*OPAP*) is generated afterwards in order to constrain the vehicle to stay within the road range. The *OPIDP* and *OPAP* are then used as the reference set-point to an optimization algorithm based on the Covariance Matrix Adaptation Evolution Strategy (CMA-ES) that computes the corresponding low-level control sequence $\mathbf{u}(\mathbf{t}) = (v(t), \delta(t))^T$ in order to achieve the safe evasive action. Indeed, instead of planning and re-planning the trajectory that must be followed by the ego-vehicle, it is imposed on the ego-vehicle to stay within the boundaries of the reference S_{PIDP}.

To summarize, in addition to the properties (1 and 2) defined for the single hypothesis use case, the multi-hypothesis optimal profiles ensures that the resulting behavior of the vehicle: favors comfortable acceleration in the lateral and longitudinal direction, allows higher maneuverability and smooth changes in the evasion and account for possible uncertainties in the states. Additional constraints are considered in the multi-criteria optimization such as: the jerk and the suppression of high curvature rates to guarantee a further smooth and comfortable trajectory.

D. Multi-objective function

The optimal sequence $\mathbf{u}(\mathbf{t}) = (v(t), \delta(t))^T$ is defined as the one that minimizes a global function that combines both the error objective functions related to OPIDP and OPAP and is

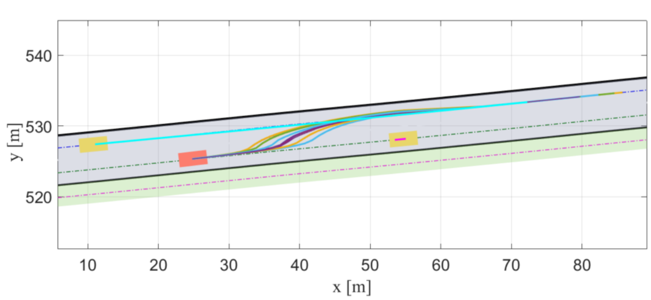

Fig. 5: Multi-Hypothesis lane change left evasive trajectories

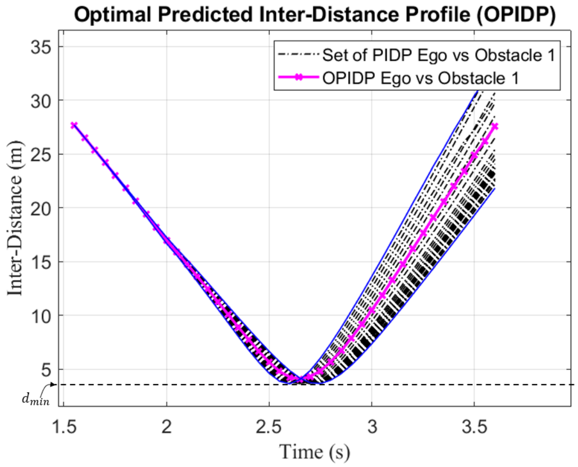

Fig. 6: Set of Predicted Inter-Distance Profile (S_{PIDP})

defined as the following:

$$J[u(t)] = \int_{t_0}^{t_0+T_h} F[u(t)]dt \tag{2}$$

with

$$F[u(t)] = \sum_{i=1}^{n_{obstacles}} \left(w_{d_i}\ f_{OPIDP_i} + \ w_{a_i}\ f_{OPAP_i} \right) \tag{3}$$

Where, for the concerned ego/obstacle –vehicle pair:

- f_{OPIDP} is the absolute value of the error between the reference OPIDP and the expected inter-distance when applying the control sequence $u(t)$ at a given time.
- f_{OPAP} is the absolute value of the error between the reference OPAP and the expected inter-angle when applying the control sequence $u(t)$ a given time.

The time t_0 is the current time, T_h is the time horizon in the interval $[t_0,\ T_{ch}]$ and i is the obstacle's Id number. Proper normalization of the objectives has been performed so that the ranges/values of each objective could be modulated/balanced between them. $w_d \in\ \mathbb{R}^+$ and $w_a \in\ \mathbb{R}^+$ are the weighting coefficients related to the objective functions f_{OPIDP} and f_{OPAP}. The weighted sum method has been used in order that each objective has its own weight w.r.t. the other sub-objective. Precise analysis of the appropriate balance between each sub-criterion or even the on-line updating of these parameters will be investigated in future work.

E. Formalization of the objective function f_{OPIDP}

The motion of the ego-vehicle is described by a tricycle model. In what follows $\mathbf{X} = \{x, y, \theta\}$ is the state vector with (x, y) the vehicle's position and θ its orientation, v and δ are output of the control law representing the velocity and the steering angle respectively, l_b is the wheel-base of the vehicle.

Based on Euler's Method to solve first order differential equation with a given initial value, we can write:

$$\begin{cases} x(t+h) = x(t) + h\ v(t)\ cos(\theta(t)) \\ y(t+h) = y(t) + h\ v(t)\ sin(\theta(t)) \\ \theta(t+h) = \theta(t) + h\ v(t)\ tan(\delta(t))/l_b \end{cases} \tag{4}$$

with $t \in [t_0, T_{pred}]$ and h the time step size.

The motion of the surrounding obstacle-vehicles is assumed to be rectilinear, uniformly accelerated. However, its dynamic can be adapted to perform other behaviors without changing the conducted reasoning. Indeed, its dynamic can be linear during the defined control horizon T_{ch} and then change and be re-adapted for the next T_{ch}. It is described by the following equations:

$$\begin{cases} x_{obs}(t+h) = x_{obs}(t) + \dfrac{1}{2} a_{x_{obs}}(t)h^2 + v_{x_{obs}}h \\ y_{obs}(t+h) = y_{obs}(t) + \dfrac{1}{2} a_{y_{obs}}(t)h^2 + v_{y_{obs}}h \end{cases} \tag{5}$$

With (x_{obs}, y_{obs}) the obstacle-vehicle's position, $(v_{x_{obs}},\ v_{y_{obs}})$ the speed components, $(a_{x_{obs}},\ a_{y_{obs}})$ the acceleration components and with $t \in [t_0, T_{pred}]$ and h the time step size.

The formalization of an inter-distance prediction profile can be defined as the function $p(t+h)$ over the interval $t \in [t_0, T_{pred}]$:

$$\begin{aligned} p(t+h) &= \left(\left(x(t+h) - x_{obs}(t+h)\right)^2 + \left(y(t+h) - y_{obs}(t+h)\right)^2 \right)^{1/2} \\ &= \Bigg(\Big(x(t) + h\ v(t)\ cos\big(\theta(t) + h\ v(t)\ \frac{tan(\delta(t))}{l_b}\big) - x_{obs}(t) \\ &\quad - h^2 \frac{1}{2} a_{x_{obs}}(t) - h\ v_{x_{obs}}(t)\Big)^2 + \Big(y(t) + \ h\ v(t)\ cos\big(\theta(t) \\ &\quad + \ h\ v(t) \frac{tan(\delta(t))}{l_b}\big) - y_{obs}(t) - h^2 \frac{1}{2} a_{y_{obs}}(t) - h\ v_{y_{obs}}(t)\Big)^2 \Bigg)^{1/2} \end{aligned} \tag{6}$$

By analyzing the following formalization given in equation (6), one can see that it highlights the needed sequence $\mathbf{u(t)}$. This formulation allows to have convenient way to define for each ego-vehicle/obstacle combination, an error objective function of the inter-distance between the reference OPIPD and the prediction $p(t+h)$ when applying the control sequence $\mathbf{u(t)}$ at a given time, and is defined as follows:

$$f_{OPIDP}(t) = \mid p(t+h) - OPIDP(t+h) \mid \quad \text{for } t \in [t_0,\ T_{pred}] \tag{7}$$

F. Formalization of the objective function f_{OPAP}

The formalization of an angular prediction profile, defined as function $\theta(t+h)$ (cf. equation (4)) over the interval $t \in [t_0, t_{pred}]$, that highlights the concerned control sequence $u(t)$ is then:

$$\theta(t+h) = \theta(t) + \frac{h\ v\ tan(\delta)}{l_b} - \theta_{obs} \tag{8}$$

With θ_{obs} the heading of the concerned obstacle-vehicle. Similarly to the OPIDP, the strategy is to minimize the absolute value of the error between the reference OPAP and the prediction $\theta(t+h)$ when applying the control sequence $\mathbf{u(t)}$ at a given time. The used error objective function is defined as:

$$f_{OPAP}(t) = \mid \theta(t+h) - OPAP(t+h) \mid \quad \text{for } t \in [t_0,\ T_{pred}] \tag{9}$$

G. Constraints definition

In addition to the longitudinal and lateral parameters used in the generation of the profiles, the optimal sequence must minimize the function described by equation (2) and at the same time obey to a set of defined constraints. These constraints result from the limits of the vehicle kinematics and dynamics and also to favor more comfortable and smooth evasive maneuver. The steering input angle is limited by the steering geometry of the vehicle concerning the steering lock angle and the steering rate of change as we aim at minimizing J and punishing high curvature rates to achieve smooth trajectories, thus:

$$\begin{gathered} -\delta_{max} \leq \delta(t) \leq \delta_{max} \\ |\dot{\delta}(t)| \leq \dot{\delta}_{max} \end{gathered} \tag{10}$$

A jerk term is used to further smoothness the trajectory by dampening rapid changes in acceleration, so:

$$\dot{a}_{\min} \leq \dot{a}(t) \leq \dot{a}_{\max} \tag{11}$$

H. Solving the optimization problem based on CMA-ES

This optimization problem is solved using an evolutionary algorithm called the Covariance Matrix Adaptation Evolution Strategy (CMA-ES) [17] that is able to reach a global optimum in few generations. Very few modifications have been introduced to the original algorithm as the strength of the CMA-ES is that it does not require a tedious parameter tuning and the choice of internal parameters of the strategy is not left to the user except for population size. The algorithm takes as input the defined multi-objective function, the initial velocity/steering configuration, the weights and constraints thresholds. The proposed strategy allows to increase the degrees of freedom concerning the maneuverability of the vehicle (ability of the system to generate variable linear velocity and steering angle solutions), ensure smooth changes during the evasive maneuver, and ensuring the safety of the system and respects as much as possible the passengers' comfort.

IV. Simulation Results

To evaluate the presented approach in simulation, the authors have developed a simulator using MATLAB/Simulink on a computer with an Intel Core i5, CPU: 1.4 GHz, RAM:8GB. For the different simulations shown below (See. Simulation Video : https://shorturl.at/psAQ4), it is considered what follows:

- The perceived scene is constituted of four vehicles in a two-lane highway (cf. Fig. 7c). On the right lane, the ego-vehicle and the ahead obstacle-vehicle 1 O_1 and on the left lane, obstacle-vehicle 2 O_2 in front and obstacle-vehicle 3 O_3 behind.
- The initial velocities of the vehicles are given by: $V_{ego_{max}} = 35m/s$, $V_{O1} = 12m/s$, $V_{O2} = 20m/s$ $V_{O3} = 30m/s$.
- The parameters taken for the optimization are as follows: the weights $w_{a_1} = 0.4$, $w_{d_{1,3}} = (0.3, 0.3)$ and the number of generation $n = 5$.
- Table I summarizes the relative parameters referred by this study.

Parameters (units)	Bounds or value	Parameters (units)	Bounds or value
$a_{max,accel}(m/s^2)$	$[3.5, 7.84]$	$a_{min,accel}(m/s^2)$	$[1, 3.5]$
$a_{max,brake}(m/s^2)$	$[-7.84, -3.5]$	$a_{min,brake}(m/s^2)$	$[-3.5, -1]$
$t_s(s)$	$[0.5, 3.5]$	$a^{lat}_{min,brake}(m/s^2)$	$[-2, -1]$
$a^{lat}_{max,accel}(m/s^2)$	$[1, 5.88]$	ρ	0.2
mu	0.2	$\lvert\delta_{max}\rvert$	$\pi/6$

TABLE I: Summary of parameters

A. Comparison between single hypothesis and multi-hypothesis

In what follows, we have selected a dangerous scenario where the obstacle-vehicle 1 in front suddenly brakes and a fast obstacles-vehicle 3 is driving on the left lane. Following the reasoning and procedure for computing the evasive maneuver (cf. Fig. 2), the single hypothesis for lane change left is performed (as shown in Fig. 7(a)) as an emergency braking is not feasible. However, we can directly notice that the profile of obstacle-vehicle 3 overrides the minimal distance requirements d_{min} therefore the multi-hypothesis is used in this case (cf. Figure 7(b)) in order to escape the dangerous situation. Indeed, this kind of situation may necessitate maneuvering up to the vehicle's handling limits. A shoulder lane or any other space alternative is not always available and the only optimal solution in this case is to quickly get into the adjacent lane (cf. Figure 7(c)). The CMA-ES computes then the appropriate control sequence that allows to follow as accurately as possible the defined profiles. The average computation time for the optimization part is calculated to be $t = 0.09$.

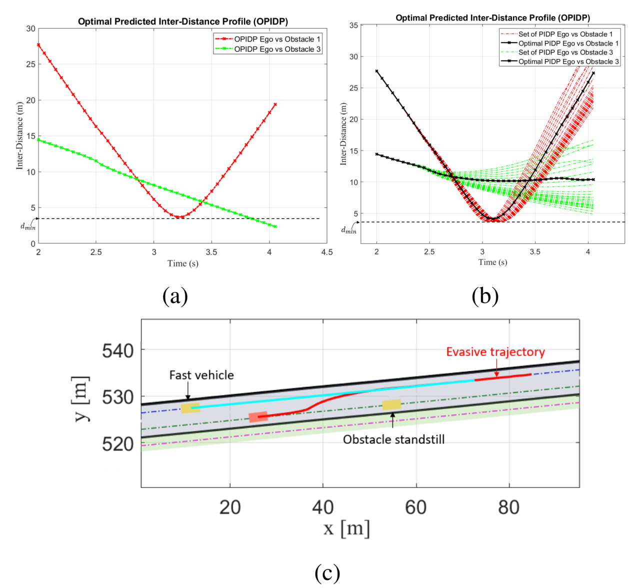

Fig. 7: Comparison between Single Hypothesis (a) and Multi-Hypothesis Prediction Profiles (b) in emergency situation (c)

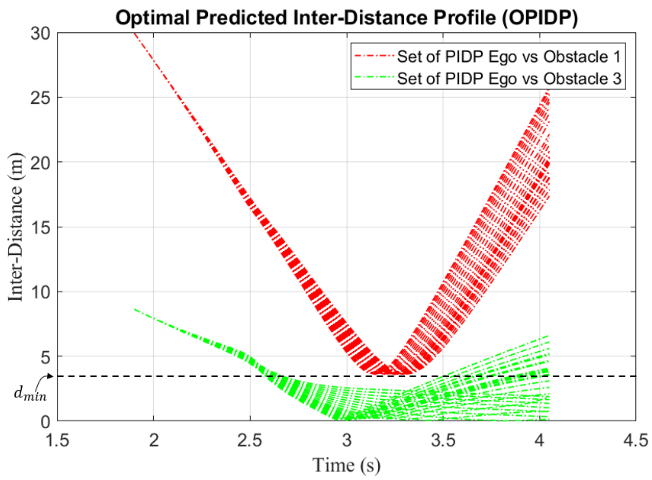

Fig. 8: Multi-Hypothesis lane change left evasive trajectories

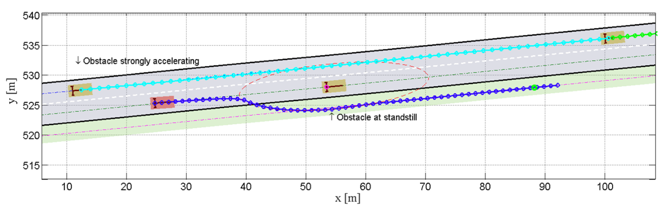

Fig. 9: Evasive trajectory involving swerving to the shoulder lane for Scenario 2

B. *Emergency Stopping Lane*

In a similar kind of scenario, in addition to the situation described in the first scenario, we simulated a sudden acceleration of the obstacle-vehicle 3 coming from behind in the left lane. Following the reasoning and procedure for computing the evasive maneuver (cf. Fig. 2), we run the multi-hypothesis process and we can see that none of the hypotheses is feasible as the profiles overrides the minimal distance requirements (cf. Fig. 8). Which leads the system to swerve to the Emergency Stopping lane (cf. Fig. 9) in order to avoid a collision, when the ESL is available otherwise collision mitigation solutions should be considered. The CMA-ES computes then the appropriate control sequence that allows to follow as accurately as possible the defined profiles. The average computation time for the optimization part is calculated to be $t = 0.07$.

V. Conclusion

This paper proposes a multi-hypothesis evasive strategy able to cope with any dynamic traffic situation. It is based on: a Sequential Decision Networks for Maneuver Selection and Verification (SDN-MSV) that calculates discrete evasive decision maneuver and an exhaustive evasive trajectory generation that takes into account the evasive decision and considers multi-hypothesis kinematic and dynamic configuration. Furthermore, a multi-criteria optimization is performed that takes into account the mentioned exhaustive process and is able to generate the corresponding low-level control that allows the ego-vehicle to pursue the safest and most comfortable advised collision-free evasive maneuver. At the same time, the algorithm minimizes jerk, punish high acceleration and curvature rate to provide enhanced comfort for passengers. Several simulation results show the good performance of the overall proposed evasive strategy. An important area of improvement would be to perform quantitative assessment on the proposed method and to optimize the computation time and the prediction horizon that are among the main works to be done in near future.

References

[1] S. Shalev-Shwartz, S. Shammah, and A. Shashua, "On a formal model of safe and scalable self-driving cars," *arXiv preprint arXiv:1708.06374*, 2017.

[2] C. Pek and M. Althoff, "Computationally efficient fail-safe trajectory planning for self-driving vehicles using convex optimization," in *2018 21st International Conference on Intelligent Transportation Systems (ITSC)*, pp. 1447–1454, 2018.

[3] M. Althoff and J. M. Dolan, "Online verification of automated road vehicles using reachability analysis," *IEEE Transactions on Robotics*, vol. 30, no. 4, pp. 903–918, 2014.

[4] S. Mitsch, S. M. Loos, and A. Platzer, "Towards formal verification of freeway traffic control," in *Proceedings of the 2012 IEEE/ACM Third International Conference on Cyber-Physical Systems*, pp. 171–180, IEEE Computer Society, 2012.

[5] C. Pek, P. Zahn, and M. Althoff, "Verifying the safety of lane change maneuvers of self-driving vehicles based on formalized traffic rules," in *2017 IEEE Intelligent Vehicles Symposium (IV)*, pp. 1477–1483, 2017.

[6] X. Huang, M. Kwiatkowska, S. Wang, and M. Wu, "Safety verification of deep neural networks," in *International Conference on Computer Aided Verification*, pp. 3–29, Springer, 2017.

[7] J. Funke, M. Brown, S. M. Erlien, and J. C. Gerdes, "Collision avoidance and stabilization for autonomous vehicles in emergency scenarios," *IEEE Transactions on Control Systems Technology*, vol. 25, no. 4, pp. 1204–1216, 2016.

[8] K. Hirsch, J. Hilgert, W. Lalo, D. Schramm, and M. Hiller, "Optimization of emergency trajectories for autonomous vehicles with respect to linear vehicle dynamics," in *Proceedings, 2005 IEEE/ASME International Conference on Advanced Intelligent Mechatronics.*, pp. 528–533, 2005.

[9] S. Magdici and M. Althoff, "Fail-safe motion planning of autonomous vehicles," in *2016 IEEE 19th International Conference on Intelligent Transportation Systems (ITSC)*, pp. 452–458, 2016.

[10] D. Iberraken, L. Adouane, and D. Denis, "Reliable risk management for autonomous vehicles based on sequential bayesian decision networks and dynamic inter-vehicular assessment," in *2019 IEEE Intelligent Vehicles Symposium (IV)*, pp. 2344–2351, June 2019.

[11] D. Iberraken, L. Adouane, and D. Denis, "Multi-level bayesian decision-making for safe and flexible autonomous navigation in highway environment," in *2018 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, pp. 3984–3990, Oct 2018.

[12] D. Iberraken, L. Adouane, and D. Denis, "Safe autonomous overtaking maneuver based on inter-vehicular distance prediction and multi-level bayesian decision-making," in *2018 21st International Conference on Intelligent Transportation Systems (ITSC)*, pp. 3259–3265, Nov 2018.

[13] S. Russell and P. Norvig, *Artificial Intelligence: A Modern Approach*. Pearson, 2016, Pearson, 2016.

[14] S. E. Lee, E. Llaneras, S. Klauer, and J. Sudweeks, "Analyses of rear-end crashes and near-crashes in the 100-car naturalistic driving study to support rear-signaling countermeasure development," *DOT HS*, vol. 810, p. 846, 2007.

[15] R. De Iaco, S. L. Smith, and K. Czarnecki, "Universally safe swerve manoeuvres for autonomous driving," *arXiv preprint arXiv:2001.11159*, 2020.

[16] S. Liu, X. Wang, O. Hassanin, X. Xu, M. Yang, D. Hurwitz, and X. Wu, "Calibration and evaluation of responsibility-sensitive safety (rss) in automated vehicle performance during cut-in scenarios," *Transportation research part C: emerging technologies*, vol. 125, p. 103037, 2021.

[17] N. Hansen, "The CMA evolution strategy: A tutorial," *arXiv preprint arXiv:1604.00772*, 2016.