Hybrid Energy Management Strategy based on Fuzzy Logic and Optimal Control for Tri-actuated Powertrain System

Nadir Ouddah and Lounis Adouane

***Abstract*—In this paper, an efficient Energy Management Strategy (EMS) of a specific multi hybrid plugin electric bus is designed and validated using high fidelity TruckMaker software simulation. The studied bus is equipped with a tri-hybrid powertrain in which traction torque is produced by three distinct energy sources (internal combustion engine (ICE), hydraulic accumulator and battery). To manage the complex operation of this hybrid powertrain smoothly and efficiently, an EMS composed of two control layers combining fuzzy logic and adaptive optimal control is proposed. The main purpose of this control strategy is the coordination of these multiple energy sources while minimizing the fuel consumption and ensuring smooth torque transitions between the motors. This last-mentioned benefit is quite important since smooth torque transitions helps to reduce power loss in the hydraulic system and ensure reliability of the powertrain. In addition, the proposed strategy is designed so that it respects the intrinsic constraints of the powertrain components and to deal with the uncertainties on the driving conditions when controlling the hybrid powertrain system.**

***Index Terms*—Hybrid electric urban bus, Tri-hybrid powertrain control, Energy management strategy, Hybrid control scheme, Fuzzy logic, Optimal control.**

I. Introduction

Since electric vehicles are still struggling to compete with conventional vehicles due to their low autonomy and the excessive cost of their batteries, plug-in hybrid electric vehicles (PHEV) are gaining more popularity. In fact, PHEV turns out to be the most appropriate solution available currently to deal with economic and environmental concerns related to the transportation systems. A hybrid vehicle combines, by definition, multiple energy sources that complement each other and guarantee efficient propulsion of the vehicle. Generally, at least two motors are associated with the mechanical transmission elements to ensure the traction of the vehicle. The arrangement of these elements defines the vehicle architecture. There are many possible powertrain architectures such as series hybrid, parallel hybrid or series-parallel hybrid powertrain configurations [1]. The advantage of the hybridization of powertrains is to overcome the two main drawbacks of Internal Combustion Engines (ICE) that are the low energy efficiency and the power irreversibility which makes the engine unable to retrieve the energy incurred during braking. Hybridization will therefore draw on the strengths of different types of engines by combining the excellent efficiency and reversibility of electric motors with the high energy density of fossil fuels which guarantees the autonomy, limits the vehicle weight and reduces refueling time.

The presence of additional power sources in the HEV introduces additional degrees of freedom in controlling the powertrain, since at each time the driver's power request can be delivered by either one of the on-board energy sources or their combination. The additional degrees of freedom can be leveraged to reduce fuel consumption and pollutant emissions and also to optimize other possible cost such as battery life [2]. However, controlling HEV raises new problems to find the most efficient way of deciding the power distribution between the power sources. This task is performed by the EMS which is the highest control level of the powertrain's control strategy [3]. The main proposed approaches of the EMSs could be classified into two categories: rules-based and optimization based EMSs, as shown in Fig. 1. Rules-based strategies have been traditionally used by automobile manufacturers because of their simplicity and effectiveness in real-time supervisory control. They could be further classified as deterministic rules-based and fuzzy rules-based EMSs [4]. All rules-based strategies are based on intuitive control approach which is capable to translate engineers knowledge and experience into corresponding results, but they require a careful calibration of the parameters [5]. It is to be noted that research communities focus on using fuzzy logic. The main idea of fuzzy logic energy optimization strategies is to leveling the operation points of the ICE onto its high efficiency curve with the complement energy supplied by batteries to increase the ICE efficiency and decrease emission.

The optimization-based EMSs rely on the use of a model based formulation of the energy management problem. These methods can be divided into numerical and analytical approaches. In numerical optimization methods like dynamic programming [6], the global optimum is found numerically under the assumption of full knowledge of the future driving conditions. Unfortunately, the results obtained through dynamic programming cannot be implemented directly due to its high computational demands. To overcome this problem, approximated dynamic programming [7] and stochastic dynamic programming [7], [8] had been proposed as solutions. Analytical optimization methods, on the other hand, use a mathematical problem formulation to find an analytical

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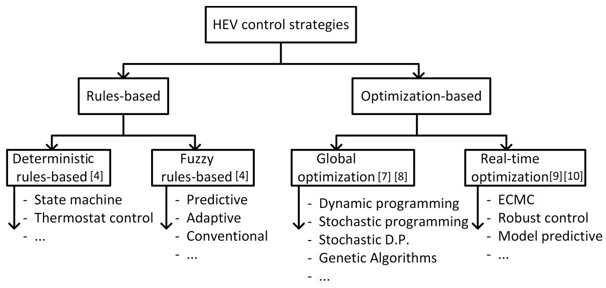

Fig. 1. Classification of hybrid vehicles control strategies.

solution that makes the numerical solution faster than the purely numerical methods. Within this category, Pontryagin's Minimum Principle (PMP) based EMS is introduced as an optimal control solution [9]. This approach can only generate an optimal solution if implemented offline. For online implementation, Equivalent Fuel Consumption Minimization (ECMS) methods that lead to suboptimal solutions have been proposed for HEVs [10] [11]. ECMS is based on instantaneous optimization. Therefore, it is suitable to be implemented in real-time. Model predictive control based methods have been also applied to solve online the energy management problem [12]. One of the main drawbacks of this approach is the high computational power required to calculate the optimal power split at each sample time.

This paper details the development of a control strategy to optimize the power distribution and to manage the global operation in a plug-in hybrid bus actuated by three distinct types of power. A major challenge in controlling such tri-hybrid powertrain is the coordination of these multiple energy sources while optimizing the whole powertrain operation. This implies to minimize at the same time: motors energy consumption, power losses in the hydraulic system and torque jolting during modes switching phases in order to avoid mechanical fatigue of the powertrain. Depleting rate of the battery in such system is considerably affected by the driving conditions, and must also be controlled properly in order to ensure a sufficient hybrid operation time of the powertrain. The aforementioned conventional control strategies do not offer satisfying performance to deal with this multi-objective problem. To address this control problem, an overall control scheme, composed of two control layers combining an adaptive optimal control and fuzzy logic strategy is proposed in this paper. It was investigated in [11] an online EMS based on PMP, which allows us to adapt the control parameters of the proposed PMP formulation according to both: the current battery SOE (State Of Energy) and to the uncertainties on the knowledge of the HEV driving cycle. The main interest of using PMP, as shown in [11], is to provide a rigorous and clear mathematical formulation for optimizing the hybrid operation mode (cf. description of mode 4 in Section II) of the studied plug-in hybrid powertrain. This mathematical analysis used in such control approach is also particularly suitable for embedded control applications because of its low computational demands [9] [10] [11]. Moreover, the aforementioned investigations on analytical control approaches has shown that PMP based optimization is probably the most appropriate candidate since it can guarantee, under given conditions, near optimality while keeping the overall methodology enough simple [13]. Based on these results, similar optimization approach has been used to design an adaptive optimal control part of the overall proposed control scheme (cf. Section III). On the other hand, a smoother operation and more control flexibility of the studied hybrid powertrain is needed especially during torque production switching phases to avoid power loses and torque jolting problems explained above. Results obtained in works dealing with similar issues such as in [14] [15] or [16] have shown that fuzzy logic control is particularly efficient for designing high performance and flexible control able to guarantee smooth operation despite HEV's dynamic states variations and environments changes. These characteristics meet perfectly the need of the targeted architecture. Thus, fuzzy logic control approach has been chosen, in combination with analytical optimization, to control the different operations modes (and their transitions) of the studied complex powertrain. The aim is to ensure the minimization of the total energy consumption of the studied HEV (with its different modes), while permitting at the same time to follow the desired battery depleting level and to have smooth transitions between the different modes of the studied tri-hybrid bus. The key contribution of this paper is in proposing a hybrid control architecture which allows at the same time to manage smoothly and efficiently all the power sources of a complex tri-hybrid powertrain and to solve the problems of torque jolts that cause power losses, mechanical fatigue of the powertrain, and driving inconvenience, while ensuring a simpler control architecture that remains easy to be implemented in an embedded controller.

The paper is structured as follows: section II describes the studied hybrid bus architecture. Section III introduces the proposed control strategy. In section IV, several simulations results are presented showing the efficiency of the proposed control strategy. Finally, conclusions and some prospects are given in the last section.

II. Tri-hybrid bus description

The aim of this section is to illustrate the architecture of the studied system, i.e., BUSINOVA hybrid bus [17] (cf. Fig. 2), developed by SAFRA. This bus is composed of an electric motor, a hydraulic motor, an internal combustion engine, a hydraulic accumulator, and a battery as the propulsion powertrain system of the vehicle. The electric motor is a 103 kW permanent magnet electrical machine from Visedo®, developed especially for heavy duty applications. It has six pole pairs and its nominal voltage is 500 V [18]. The internal combustion engine is produced by VM Motori®. It delivers a maximum torque of 340 $N.m$ at 1400 rpm and its maximum produced power is 70 kW [19]. The hydraulic motor is a Parker® V14 series with a displacement that varies between 22 and 110 cm^3 [20].

Powertrain architecture

The model of the studied hybrid bus is based on a series-parallel power-split hybrid architecture. A simple block diagram of the power flows on the bus is shown in Fig. 3.

Fig. 2. BUSINOVA hybrid bus.

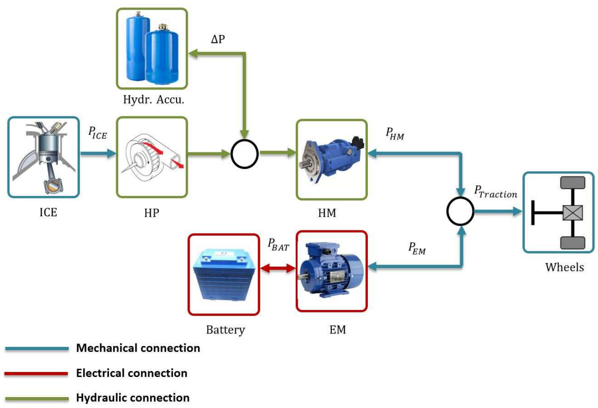

Fig. 3. Block diagram of the powertrain power flows. (ICE: internal combustion engine, HP: hydraulic pump, HM: Hydraulic motor, EM: electric motor).

The electric and hydraulic motors are both directly connected to the transmission and can ensure simultaneously or independently the traction of the bus. The electric and hydraulic motors combination allows at the same time to satisfy high torque requirements during start-up and climbing phases while eliminating the need for a big and less energy efficient electric motor, that basically can not be used at its full capacity most of the time during cruising phases.

On the other hand, the internal combustion engine is coupled with a hydraulic pump for driving the hydraulic motor. This later one is equipped with a variable displacement allowing to optimize the efficiency of the overall hydraulic system by shifting the internal combustion engine load, and thus allow it to operate close to its maximum efficiency curve. The hydraulic accumulator is mainly used to drive the hydraulic motor during start-up phases when a large amount of torque is needed.

The rotational speeds of the hydraulic motor and the electric motor are imposed by the wheels speed in proportion to the reduction ratios of hydraulic and electric motors respectively. Moreover, the rotational speed ω_{HM} and the torque T_{HM} of the hydraulic motor are expressed as a function of the rotational speed and the torque of the internal combustion engine as follows.

$$\begin{cases} \omega_{HM}\left(T_{ICE}, D_{HM}\right) = \dfrac{D_{HP}.\eta_{v_{HM}}.\omega_{ICE}}{D_{HM}.\eta_{v_{HP}}} & \text{(1a)} \\ T_{HM}\left(T_{ICE}, D_{HM}\right) = \dfrac{D_{HM}.\eta_{m_{HM}}.T_{ICE}}{D_{HP}.\eta_{m_{HP}}} & \text{(1b)} \end{cases}$$

where ω_{ICE}, T_{ICE} are respectively rotational speed and torque of the ICE, and D_{HM}, D_{HP}, $\eta_{m_{HM}}$, $\eta_{m_{HP}}$, $\eta_{v_{HM}}$, $\eta_{v_{HP}}$ are respectively the displacements, mechanical efficiency and volumetric efficiency of the hydraulic motor (HM) and the hydraulic pump (HP).

The studied hybrid bus can operate in four traction modes as detailed below:

- Mode 1 (Electric mode): internal combustion engine and hydraulic motor are stopped, the bus is powered by the energy of the battery.
- Mode 2 (Hydraulic via ICE mode/Degraded mode): when the battery is depleted, the bus is propelled exclusively by the internal combustion engine through the hydraulic pump. In this mode, ICE runs at optimum torque while electric motor (as generator) regenerates the part of the energy which is not needed for driving.
- Mode 3 (Hydraulic via accumulator mode): the bus is propelled via the transmitted power from hydraulic accumulator to hydraulic motor.
- Mode 4 (Hybrid mode): the internal combustion engine provides a complement of power to the electric motor through the hydraulic motoring system.

For security reasons, Braking modes management, which includes the repartition of braking torques between electric motor, hydraulic motor, and conventional brake, is carried out as it is demanded by the European transport regulation committee [21].

III. Proposed control strategy

Urban transportation systems of new generation are increasingly incorporating hybrid solutions. The hybrid bus studied in this paper is no exception to this trend since it is equipped with three different types of energy sources ensuring its propulsion. Improving energy economy of such modern hybrid vehicle involves the joint optimization of the powertrain's architecture and its control strategy. This latter point is the subject of the work detailed in this section.

The main challenge in controlling a hybrid powertrain is to manage the interaction between its different components in order to obtain the most efficient operation of the vehicle while respecting the physical limits of the actuators and satisfying passengers comfort constraints. In this perspective, the control architecture proposed in this paper in intended to optimize the operation mode selection of the powertrain, optimize the power split between the motors and reduce torque jolts in order to improve the reliability of the powertrain and the comfort of passengers. Taking into account the complexity of the considered powertrain architecture, a control scheme combining fuzzy logic and adaptive optimal control is proposed in the following.

A. Overall control scheme

The overall control scheme describing the proposed energy management and optimization strategy of the tri-hybrid powertrain is given in Fig. 4. The input parameters of this control architecture are: the torque set-point to be set by the driver through the accelerator pedal, bus position/location

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information as well as bus status data which includes speed information, and State Of Charge (SOC) of the battery and hydraulic accumulator. As shown in Fig. 4, these input signals are first used in the upper layer of the control scheme to generate a mode selection signal which allows to select at each moment the appropriate operating mode of the powertrain. Energy optimization is then performed in both control layer of Fig. 4. Two types of optimization strategies are used: an adaptive optimal control strategy during regular hybrid operation of the powertrain and a fuzzy logic optimization strategy which optimize energy during modes switching and ensure the coordination with the optimal control strategy. At the output of this overall control scheme, torque set-points are given respectively for the internal combustion engine, the hydraulic motor and the electric motor. These reference values are then taken into account by the motors' local controllers to produce the torque requested by the driver.

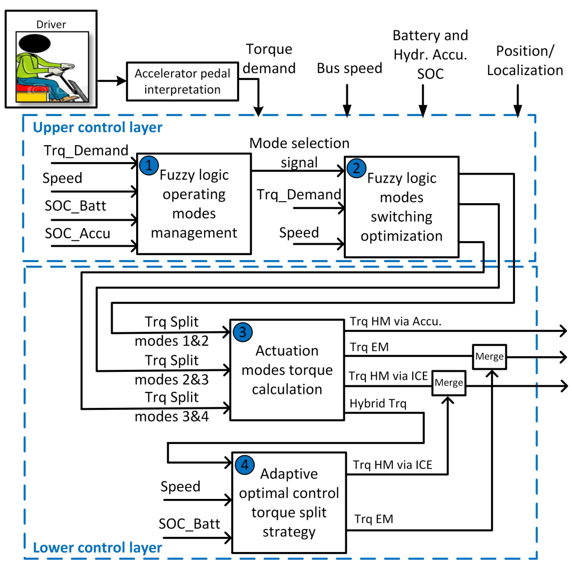

Fig. 4. Block diagram of the proposed overall control strategy.

The next subsections will be devoted to the description of the control strategies implemented on both control layers shown in Fig. 4. First, the synthesis of the fuzzy logic modes switching management and optimization strategy implemented in the upper control layer is presented in the next part of this section. The development of the optimal control algorithm implemented in the lower control layer is then detailed in the last part of this section.

B. Modes switching management and optimization

The first task that must be accomplished by the proposed control strategy is online selection of the powertrain's operating mode as well as optimization of the switching between modes when a transition is needed. The aim of this control subsystem is to determine in real time the most appropriate operating mode of the hybrid powertrain and to optimize torque switching in order to enhance fuel economy and smoothness of operation during mode transition. As shown in Fig. 4, these control tasks are carried out by two complimentary fuzzy controllers which are implemented in blocks 1 and 2 of this figure: *1) Fuzzy logic operating modes management strategy*, implemented in the upper control layer of the global control scheme, which supervises and controls the selection of the appropriate operating mode, *2) Fuzzy logic modes switching optimization strategy* implemented also in the upper layer of the control scheme for the optimization of the modes' switching.

Given the complexity of the studied tri-hybrid powertrain architecture, the use of fuzzy logic is of major interest for the realization of the operating modes management and optimization strategies. Indeed, the boundaries between the operating modes are not well defined due to the nonlinear and time-varying characteristic of the powertrain's dynamic model [22]. In addition, instead of using deterministic rules as it is the case in most commercially available HEVs, the use of fuzzy logic allows to prevent from abrupt switching between the operating modes of the hybrid powertrain which results in less power losses in the hydraulic system and less mechanical fatigue of the actuators and the transmission.

Multiple inputs single output fuzzy logic control was used to implement the fuzzy logic operating modes management strategy (cf. block 1 of Fig. 5). The inputs to this block are the requested torque signal, the measured bus speed signal as well as the SOE and the SOC status of the battery and the hydraulic accumulator respectively. The output is a mode selection signal used to determine the operating mode of the powertrain. The membership functions of these input and output variables are designed as Fig. 6. As shown in this figure, a trapezoidal shape of input/output membership functions is chosen for this fuzzy controller. In particular, the trapezoidal shape of the output membership functions helps to merge between the fuzzy logic-based strategy and the optimization strategy based on optimal control (cf. block 4) during hybrid operating phases. Mamdani (max-min) algorithm is used as fuzzy reasoning method and center of gravity (COG) defuzzification is then carried out to calculate the value of the mode selection signal at the output of the fuzzy logic operating modes management controller (cf. block 1).

Afterwards, the mode selection signal is used as an input for the fuzzy logic modes switching optimization block (cf. block 2 of Fig. 5). The other inputs of this block are the requested torque and the measured bus speed signals. The outputs of this block are the modes switching coefficients used to calculate the torque split between the operating modes during mode transition phases. The membership functions of input and output variables of the fuzzy logic modes switching optimization controller are designed as Fig. 7. As one can see, Triangular and trapezoidal functions are used for their simplicity. As before, Mamdani inference algorithm and center of gravity defuzzification method are used to calculate the modes switching coefficients at the output of the switching optimization block.

For both modes management and switching optimization controllers, fuzzy rules are constructed based on background knowledge and experience of control of the tri-hybrid powertrain. Actually, functional tests are first performed on the real bus to determine the efficiency, response time and max-

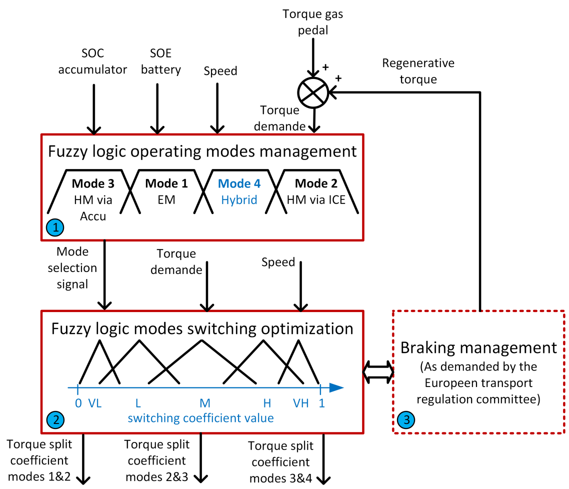

Fig. 5. Block diagram of the fuzzy logic operating modes management and optimization strategy.

imum torque characteristics over the entire operating range of each actuator composing the studied tri-hybrid powertrain. In addition, operating configurations that may cause failures or excessive vibrations and noise are identified and taken into account. Then, based on the observed operating characteristics and while taking into consideration in real time: the state of charge of the battery and the hydraulic accumulator; the availability of the actuators as well as some other parameters related to the mass of the bus and the road slope; the fuzzy rules for selecting the appropriate operating modes and the optimal torque produced by each actuator are determined in accordance with the control objectives. More precisely, the rules design is carried out so as to optimize a global optimization criterion representing a compromise between energy efficiency and smooth operation of the hybrid powertrain. Tables I and II present examples of fuzzy logic rules of the modes management and the switching optimization controllers.

The objective of the modes switching optimization strategy is twofold: on one hand, to act effectively on reducing torque jolts and energy consumption during modes switching phases thanks to dedicated accurate torque control during the switching, and on the other hand, to make link with the optimal control based optimization strategy, which will be presented in section III-C, by managing mode switching from and toward the hybrid propulsion mode in which the torque split between the motors is ensured by the optimal control strategy implemented in the lower control layer.

TABLE I
FUZZY LOGIC RULES OF THE MODES MANAGEMENT CONTROLLER (L: LOW, M: MEDIUM, H: HIGH).

Torque demand	Bus speed	SOC Bat.	SOC Accu.	Mode
M, H	L	L, M, H	M, H	Mode 3
L	L	M, H	L, M, H	Mode 1
...	...	...	...	...
L, M	H	L	L	Mode 2

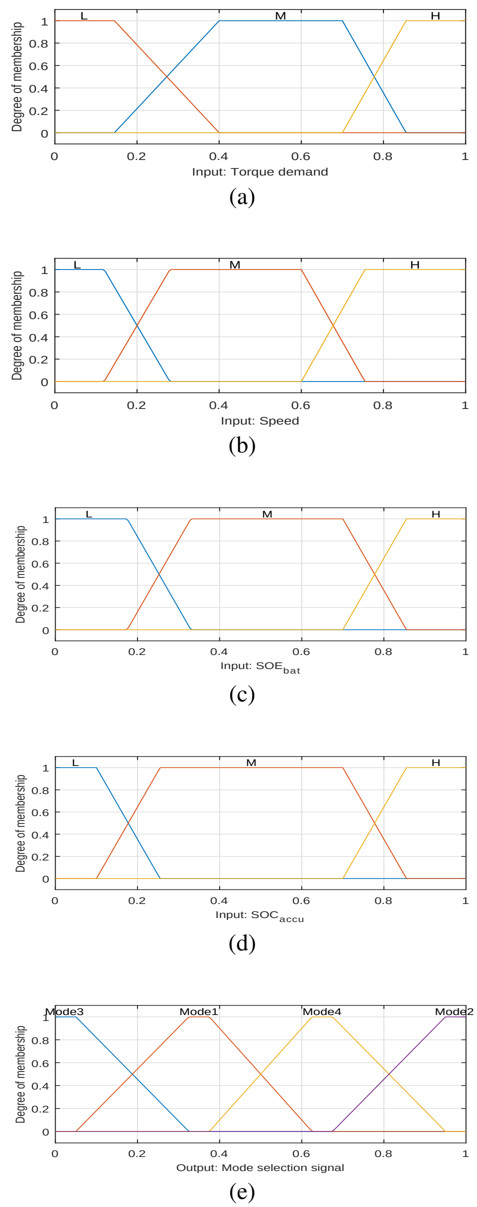

Fig. 6. Membership functions of modes management fuzzy logic controller (L: low, M: medium, H: high).

TABLE II
FUZZY LOGIC RULES OF THE SWITCHING OPTIMIZATION CONTROLLER (VL: VERY LOW, L: LOW, M: MEDIUM, H: HIGH, VH: VERY HIGH).

Selection signal	Torque demand	Bus speed	Torque split
VL	L	L, M, H	VH
VL	M	L, M, H	H
...	...	...	...
VH	M, H	H	VL

C. Hybrid operation mode optimization strategy

When the hybrid operating mode is selected, an energy optimization control strategy (cf. block 4 of Fig. 4) is used to calculate the optimal power split between the electric motor and the hydraulic motor (driven by the ICE via the hydraulic

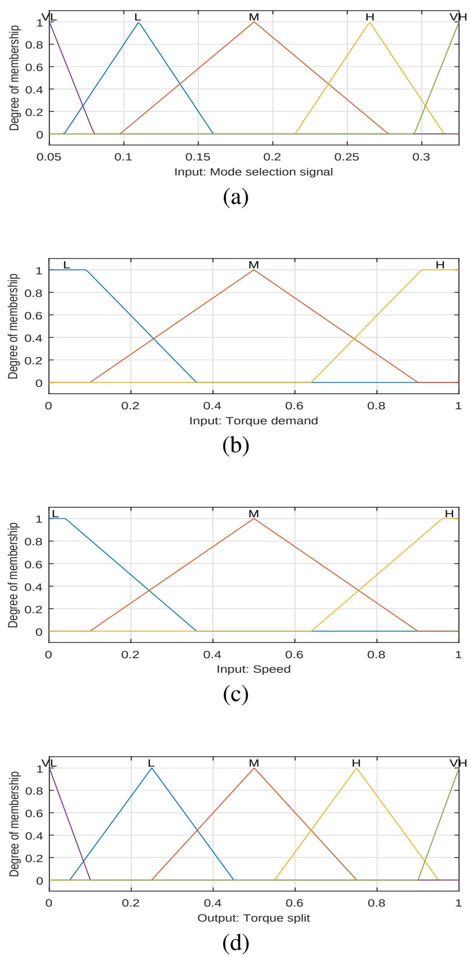

Fig. 7. Membership functions of modes switching optimization fuzzy logic controller (VL: very low, L: low, M: medium, H: high, VH: very high).

pump). A detailed block diagram of this control subsystem, which is implemented in the lower control layer, is illustrated in Fig. 8.

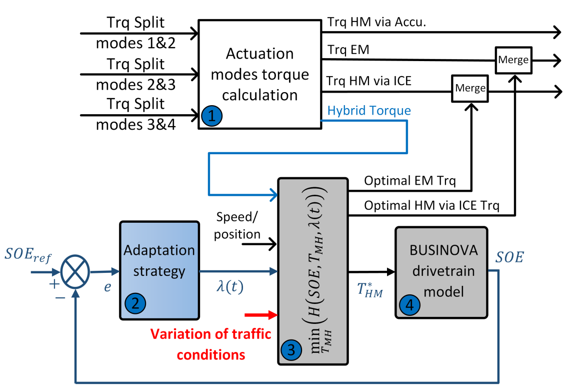

Fig. 8. Block diagram of the hybrid operation mode optimization strategy.

The fusion between this controller and the modes switching optimization controller allows to combine their advantages in order to obtain at the same time a smooth switching between the powertrain's operating modes, and an improved fuel economy.

1) Optimization problem formulation: The main objective of the optimal control strategy proposed in this section is to find, at each sample time during hybrid operating mode, the optimal value of the control input that minimizes a cost function representing the power consumption of the powertrain without sacrificing the bus drivability. The control input in this case is the power split between the electric motor and the hydraulic motor (represented by the power produced by one of either motors). The amount of residual energy of the battery represented by the estimation of the battery State Of Energy SOE is the main dynamic state. The state equation connecting the variation of the battery's remaining energy to the control variable of the system can be represented by:

$$\dot{x}(t) = \frac{d\ SOE(t)}{dt} = -\frac{P_{BAT}}{E_{max}} = -\frac{P_{EM}}{\eta E_{max}} \quad (2)$$

Depending on whether the battery is in discharging phase ($\dot{SOE} \leq 0$) or in charging phase ($\dot{SOE} \geq 0$), η is defined as follows [23]:

$$\eta = \begin{cases} \eta_{BAT} & if \quad in\ discharging\ phase \\ 1/\eta_{BAT} & if \quad in\ charging\ phase \end{cases} \quad (3)$$

Equation 2 is obtained from the battery internal resistance model [23]. In this equation, E_{max} is the maximum energy that can be stored in the battery, η_{BAT} is the efficiency of the battery, P_{BAT} is the power delivered by the battery and P_{EM} is the power consumed by the electric motor to produce torque T_{EM} at speed ω_{EM}.

The minimization of the cost function must be done under a certain number of constraints. In fact, the powertrain components dimensioning imposes minimum and maximum limits on the exchanged powers. These limits form the following constraints.

- The internal combustion engine and electric motor have limited operating ranges. Therefore, provided or absorbed torques must be comprised between minimum and maximum limits.

$$T_{EM}^{min} \leq T_{EM}\ (t) \leq T_{EM}^{max} \quad (4)$$

$$T_{HM}^{min}\left(T_{ICE}^{min}, D_{HM}\right) \leq T_{HM} \leq T_{HM}^{max}\left(T_{ICE}^{max}, D_{HM}\right) \quad (5)$$

The maximum and minimum torque limits of the internal combustion engine and electric motor vary according to the variation of the system's operating point (torque-speed). Data tables along with linear interpolations are therefore used to determine their values at each time.

- The instantaneous power demand of the powertrain should always be satisfied, which results in,

$$\rho_1 T_{HM}(T_{ICE}, D_{HM}) + \rho_2 T_{EM}(t) - T_{wheel}(t) = 0 \quad (6)$$

where ρ_1 and ρ_2 are the gearbox' reduction ratios of hydraulic and electric motors respectively.

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Compared with energy management problem formulation for charge sustaining HEV [24], there is no sustainability constraint on the final SOE for plug-in HEV allowing the charge depleting operation. Thus, the energy consumed on the entire cycle does not come exclusively from the fuel since most of the available electrical energy is supplied from the grid. This implies that the cost function must take into account all the energy sources used to ensure the traction of the bus. That is why the cost function J to be minimized over the time interval $[t_i,\ t_f]$ is defined based on the total electric and fuel energy consumed by the vehicle as follows.

$$J{=}\int_{ti}^{tf} P_F\left(u\left(t\right)\right)+P_{BAT}\left(u\left(t\right)\right)dt \tag{7}$$

where P_F is the instantaneous power of the fuel (engine power input). It is commonly expressed in terms of the fuel flow rate $\dot{m}_f$ and the lower heating value of the fuel ($Q_{LHV} = 43MJ/kg$) using the formulation given in equation 8 [24].

$$P_F\left(u\left(t\right)\right)=\dot{m}_f\left(u\left(t\right)\right)\ Q_{LHV} \tag{8}$$

In this optimal control problem, the motors rotational speed is imposed by the wheels speed. Thus, only torque values can be used to decide how to split the driver's demanded power. In addition, hydraulic motor torque is used in this case as a control variable instead of electric motor torque since the two variables are linked together through equation 6.

The optimization problem is then to find the hydraulic torque that should be provided at every sample time in order to minimize the total energy consumed while checking the constraints mentioned above (cf. equations 4 to 6). To these constraints it is added a new constraint 9 which aims to limit the admissible control region in order to take into account the limits of the hydraulic motor dynamics and consequently taking into account the limits of the internal combustion engine dynamics.

$$\frac{dT_{HM}}{dt}-\xi\geq 0 \tag{9}$$

with ξ the maximum hydraulic torque variation measured over a short period of time.

To introduce constraints in the optimization problem, these are transformed into equality constraints. The constraint 9 can be rewritten as follows.

$$\frac{dT_{HM}}{dt}-\xi-\dot{\varepsilon}^2=0 \tag{10}$$

where ε is a slack variable.

By using equation 6, it is possible to rewrite the constraints 4 and 5 as a single constraint on the control variable as follows.

$$\tilde{T}_{HM}^{min}\left(T_{HM}^{min},T_{EM}^{max}\right)\leq T_{HM}\leq\tilde{T}_{HM}^{max}\left(T_{HM}^{max},T_{EM}^{min}\right) \tag{11}$$

with

$$\tilde{T}_{HM}^{min}=\max(\rho_1.T_{HM}^{min},T_{wheel}-\rho_2.T_{EM}^{max}) \tag{12}$$

$$\tilde{T}_{HM}^{max}=\max(\rho_1.T_{HM}^{max},T_{wheel}-\rho_2.T_{EM}^{min}) \tag{13}$$

It means that when the torque applied to the wheel is too significant to be only produced by the electric motor, the $\tilde{T}_{HM}^{min}$ limit imposes a minimum torque on the hydraulic motor. Additionally, $\tilde{T}_{HM}^{max}$ limit prevents the electric motor torque set-point to become less than T_{EM}^{min}.

Finally, using a 2^{nd} order approximation, the constraint 11 is written as the equivalent form given by 14,

$$-T_{HM}^2+\alpha T_{HM}+\beta=0 \tag{14}$$

with

$$\text{ff}{=}\tilde{T}_{HM}^{max}-\tilde{T}_{HM}^{min} \tag{15}$$

$$\text{fi}{=}\tilde{T}_{HM}^{max}\ .\ \tilde{T}_{HM}^{min} \tag{16}$$

2) Optimization strategy description: With the optimization problem fully defined, Pontryagin's minimum principle is used to give numerical solution. In this case, minimizing the cost function given in 7 is equivalent to minimizing the Hamiltonian function H of the system at each instant of time.

$$H\left(x\left(t\right),\ u\left(t\right),\ \lambda\left(t\right)\right)=P_F\left(\ \rho_1 T_{HM}\left(t\right),\frac{1}{\rho_1}\omega_{HM}(t)\right) \\ -\left(\frac{\lambda\left(t\right)}{\eta E_{max}}-1\right)\ P_{ME}\left(\rho_2 T_{EM}\left(t\right),\ \frac{1}{\rho_2}\omega_{\ EM}\left(t\right)\right) \tag{17}$$

where $\lambda\left(t\right)$ is the costate (or the Lagrange multiplier).

For the considered energy management problem, an extended Hamiltonian function is defined to account for the constraint 10 and 14. The additional terms are introduced using a new Lagrange multipliers (i.e., $\gamma(t)$ et $\sigma(t)$ respectively).

$$H\left(x\left(t\right),\ u\left(t\right),\ \lambda\left(t\right),\ \gamma\left(t\right),\sigma(t)\right)= \\ P_F\left(\ \rho_1 T_{HM}\left(t\right),\frac{1}{\rho_1}\omega_{HM}(t)\right)-\left(\frac{\lambda\left(t\right)}{\eta E_{max}}-1\right) \\ P_{ME}\left(\rho_2 T_{EM}\left(t\right),\ \frac{1}{\rho_2}\omega_{\ EM}\left(t\right)\right)+\gamma\left(t\right)\left(-T_{HM}^2+\alpha T_{HM}\right. \\ \left.+\beta\right)+\sigma\left(t\right)\left(\frac{dT_{HM}}{dt}-\xi\right)^2 \tag{18}$$

The optimal control law which minimizes the Hamiltonian H must satisfy the following necessary conditions for optimality:

$$\begin{cases} \frac{\partial H(t)}{\partial u(t)}=\frac{\partial H(t)}{\partial T_{HM}(t)}0 & \text{(19a)} \\ -\frac{\partial H(t)}{\partial x(t)}=-\frac{\partial H(t)}{\partial SOE(t)}\dot{\lambda}^*\left(t\right) & \text{(19b)} \\ \frac{\partial H\left(t\right)}{\partial\lambda\left(t\right)}\dot{x}^*\left(t\right) & \text{(19c)} \\ \frac{\partial H(t)}{\partial\gamma(t)}=-T_{HM}^2+\alpha T_{HM}+\beta 0 & \text{(19d)} \\ \frac{\partial H(t)}{\partial\sigma(t)}=\left(\frac{dT_{HM}}{dt}-\xi\right)^2\dot{\varepsilon} & \text{(19e)} \end{cases}$$

The costate λ is determined by 19c.

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The condition 19a determines the optimal control trajectory $T^*_{HM}(t)$. If this necessary condition is satisfied, then the optimal hydraulic torque $T^*_{HM}(t)$ must be given by equation 20.

$$T^*_{HM}(t) = \arg\min_{T_{HM} \in U} H(SOE(t), T_{HM}(t), \lambda(t)) \qquad (20)$$

where U is defined as the admissible control set.

After the hydraulic motor torque is obtained, the internal combustion engine torque and speed are calculated according to the desired speed and torque of the hydraulic motor. Thanks to the displacement tuning capability of the hydraulic motor, the internal combustion engine load can be shifted freely to operate this latter close to its maximum efficiency curve. Especially in this case, the speed of the internal combustion engine is not imposed by the wheels speed and it can be set to a nearly constant value where the engine is the most efficient. To reach this goal, the displacement of the hydraulic motor is controlled online by using equation 1a. Thereafter, the engine torque is calculated as a function of the displacement and the optimal torque of the hydraulic motor by using equation 1b.

3) Optimization strategy adaptation procedure: In this paper, it is considered that the desired final value of SOE after eight hours of driving is 17%. The working hypothesis behind this assumption is to use the maximum amount of energy that can be consumed from the battery in one day driving.

For a perfectly known driving cycle, there exists only one value of the costate λ for which the solution that minimizes the Hamiltonian H at each sample time is also the one that satisfies the terminal condition on the final value of SOE. This corresponds to the global optimal solution of the problem. However, the assumption of perfect knowledge of the driving cycle is not true in practice because of the variation of traffic conditions. An optimal speed profile can, however, be predicted for each trip of the bus based on the actual driving conditions. Indeed, buses run on the same route every day, stop invariably at similar locations and they could even have some dedicated lanes of the road in some cities which facilitates driving conditions prediction compared to other type of vehicles. Therefore, several studies have been conducted to optimize bus speed profiles [25]. With this approach in mind, the optimal speed profile is first determined offline by using an external speed profile optimization algorithm based on a predictive intelligent control [26]. Once the optimal speed profile is obtained, it is used to calculate an optimal state trajectory SOE_{ref}, which will be used online, as an input for the optimal control algorithm to guide the choice of the costate value, and thus take into account driving conditions variation. The aim is to achieve the desired final SOE value at the end of the considered driving interval despite the lack of knowledge of the driving conditions. To reach this goal, the actual SOE value is approximated to its reference value SOE_{ref} obtained from speed profile optimization. The objective here is not to track the reference SOE trajectory but to use the information about the optimized driving cycle that it contains (acceleration, braking, road slope, etc.) to adapt the costate value depending on the characteristics of the route and the new driving conditions. In this costate adaptation strategy (cf. Block 2 of Fig. 8), the value of the costate is found at each sample time according to 21.

$$\lambda(t) = \tau(t)\,\lambda_{max} + (1 - \text{ø}(t))\,\lambda_{min} \qquad (21)$$

with

$$\lambda(t)|_{\tau=0.5} = \lambda_0 = \frac{\lambda_{min} + \lambda_{max}}{2} \qquad (22)$$

λ_0, λ_{min} and λ_{max} are respectively the initial, the minimum and the maximum values of the costate λ. The costate variation range (i.e., $[\lambda_{min}, \lambda_{max}]$) is chosen sufficiently large to handle all types of uncertainties on the knowledge of the driving cycle including unplanned stops. $\tau(t)$ is a tuning parameter such as $\tau \in [0, 1]$ and τ_0 is its initial value fixed at 0.5. The problem of the evaluation of $\lambda(t)$ is therefore transferred to the evaluation of $\tau(t)$.

The parameter $\tau(t)$ is estimated in real-time using the SOE feedback as stated in equation 23.

$$\tau(t) = \tau_0 - \frac{\mu\,(SOE(t) - SOE_{ref}(t))}{\Delta SOE_{max}} \qquad (23)$$

where SOE_{ref}: the optimal SOE trajectory calculated offline, ΔSOE_{max}: the maximum amount of energy that can be consumed from the battery during the whole drive cycle, and μ: a constant calibration parameter.

The objective of the suggested formula in equation 23 is to find at each time the value of τ needed to bring back the actual SOE to its desired value SOE_{ref}. In other words, when the battery SOE has a different value from the desired SOE_{ref}, the parameter τ is modified to give priority to the use of the electric motor or to the hydraulic motor and thus it tries to discharge the battery or, on the contrary, to capture as much braking energy as possible to charge the battery.

IV. Results and Analysis

The implementation of the proposed control architecture discussed in this paper is carried out using a dedicated high-fidelity model of the hybrid bus, that was developed on TruckMaker software (cf. Fig. 9). This simulation software allow one to investigate the performance of the proposed strategy in a test platform which reproduces accurately the real operating behavior of the bus.

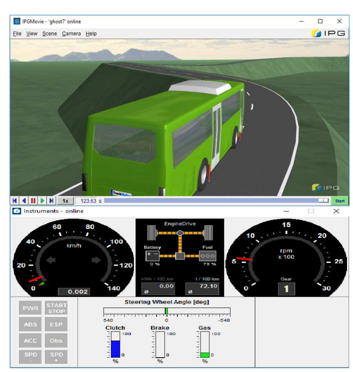

Fig. 9. TruckMaker test platform.

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Evaluation test of the proposed EMS is performed on UDC and FTP-75 normalized driving cycles which represent different usage conditions of a hybrid electric bus including urban and extra-urban driving environments. In addition, in order to assess the effectiveness of this strategy, its performance is compared to the results of a typical rules-based EMS that can be found on a classical hybrid bus. A deterministic rules-based strategy is a heuristic energy management method based on the use of a set of deterministic commutation rules to split the total power demand between the motors. In this paper, the rules-based strategy used for comparison, was developed based on the power split results from the proposed fuzzy logic/optimal control strategy. In fact, these results were analyzed to observe recurring behavior that could be replicated online using deterministic rules. The control architecture of this rules-based strategy includes two finite state machines executed in parallel as illustrated in Fig. 10.

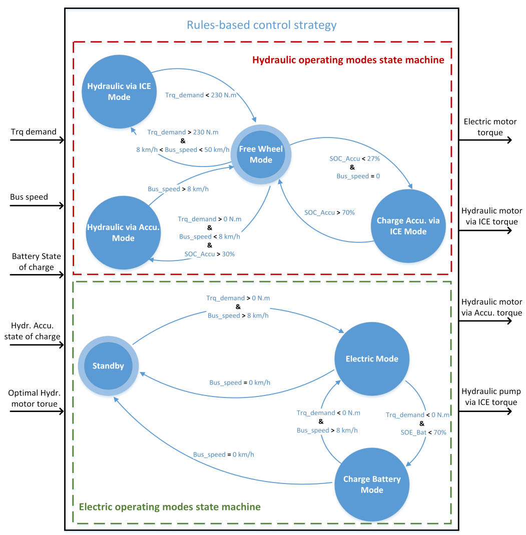
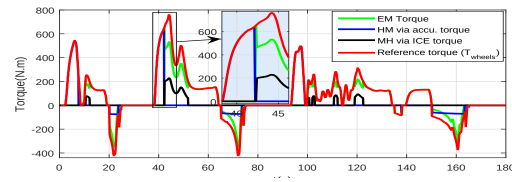


Fig. 10. Rules-based control strategy architecture.



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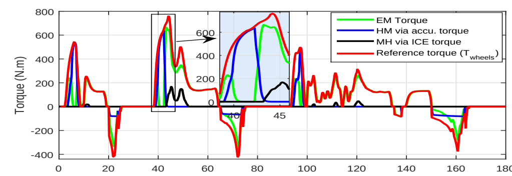

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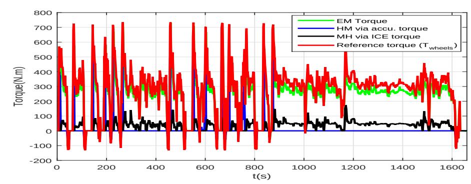
Fig. 11. Torque sharing profiles on UDC driving cycle: (a) Deterministic rules-based EMS, (b) Proposed EMS.



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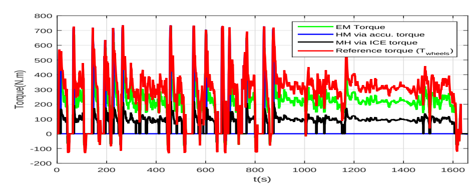

(b)

Fig. 12. Torque sharing profiles on FTP-75 driving cycle: (a) Deterministic rules-based EMS, (b) Proposed EMS.

Performance evaluation results of the proposed EMS and the deterministic rule-based EMS used for comparison are presented side by side in the following figures. During this test, initial state of charge of the battery and the hydraulic accumulator are 75% and 90% respectively. The torque split calculated by each algorithm on UDC and FTP-75 driving cycles are shown on Fig. 11 and Fig. 12.

From these figures, one can notice that the torque distribution between the motors is properly ensured with both EMSs. However, abrupt transitions and a few overshoots can be seen on the torque plots of the deterministic rules-based strategy. Otherwise, with both algorithms, the sum of the motors' produced torques corresponds to the total torque demand setpoint. In order to evaluate the smoothness of the powertrain operation using the proposed control strategy versus the rules-based control strategy, an oscillation quantifying criterion of the motors torques is defined as follows:

$$Oscillation\ criterion = \int_{0}^{t_{sim}} \left| \frac{d\ Trq}{dt} \right| .dt \qquad (24)$$

Fig. 13 and Fig. 14 represent the oscillation criterion evaluation on UDC and FTP-75 driving cycles. From these figures one can see that the torque oscillation criterion from the rules-based strategy is greater than the one obtained from the proposed control strategy and that for all the motors. Which means that motors torque ripple is noticeably reduced when using the proposed control strategy. Based on the results of figures 13 and 14, a summary of the torque smoothness gain, obtained when using the proposed control strategy instead of the reference rules-based strategy, is given in Table III.

TABLE III
TORQUE SMOOTHNESS GAIN COMPARED TO THE RULES-BASED STRATEGY.

	EM torque	**HM via Accu. torque**	**HM via ICE torque**
UDC	7.3%	13.2%	12.4%
FTP-75	6.9%	16.3%	10.6%

The curves representing the evolution of the states of charge of the battery and the hydraulic accumulator are plotted on

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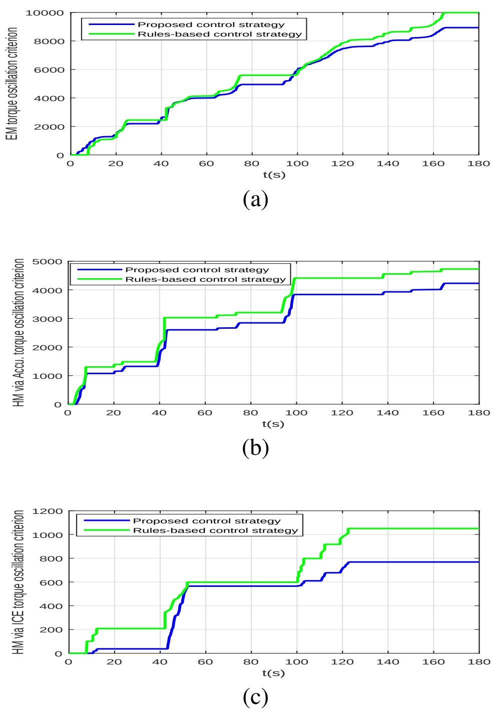


Fig. 13. Torque oscillation quantification on UDC driving cycles: (a) Electric motor torque, (b) Hydraulic motor via accumulator motor torque, (c) Hydraulic motor via ICE motor torque.

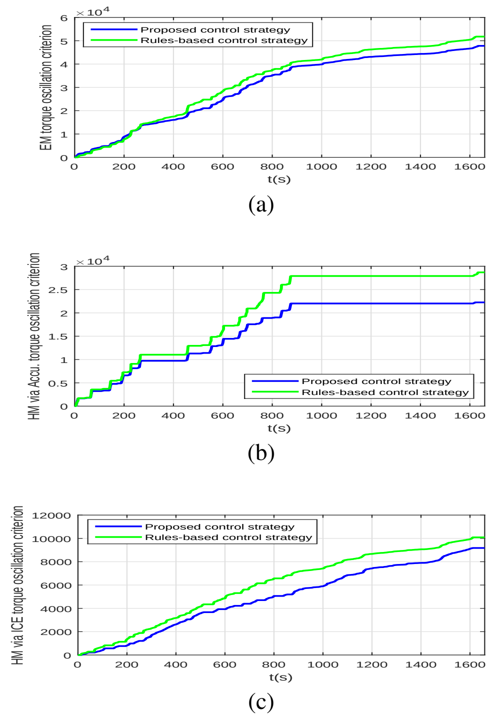


Fig. 14. Torque oscillation quantification on FTP-75 driving cycles: (a) Electric motor torque, (b) Hydraulic motor via accumulator motor torque, (c) Hydraulic motor via ICE motor torque.

figures 15 to 18. It can be seen on Fig. 15 and Fig. 17 that with the deterministic rules-based strategy, the final state of charge of the battery may have a slight difference from the desired final state of charge, because of the lack of parameters adaptation in this algorithm, which is supposed to help to take into account driving conditions variation. However, when this slight difference is interpolated over the total daily operating time of the bus, it may represent a significant drift compared to the desired battery final state of charge. In this case, the state of charge management performed in the proposed strategy is of interest to avoid a too quick battery discharge. Indeed, when the battery runs down faster than expected and the charge level reaches its minimum threshold, the degraded operating mode will be activated to recharge the battery using the internal combustion engine, which will lead to an additional cost of energy consumption induced by the low efficiency of the ICE as well as losses in the electric generator, power electronics, and battery.

On-line energy consumption evaluation results of the proposed strategy and the deterministic rules-based strategy as well as global optimal energy consumption obtained from off-line simulation of the proposed control strategy are given in Fig. 19 and Fig. 20 respectively for UDC and FTP-75 driving cycles. A summary of electrical and hydraulic average consumption, evaluated on the basis of continuous operation during 8 hours, with each control strategy is given in Table IV.

By comparing the consumption curves from the proposed on-line control strategy and the rules-based control strategy, it can be seen that the abrupt transitions, observed on the

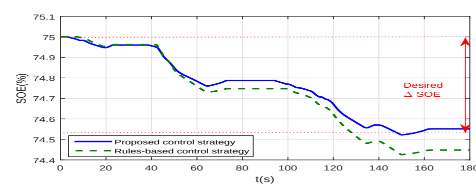
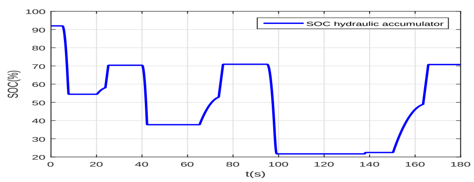


Fig. 15. Battery state of charge evolution on UDC driving cycle.

deterministic rules-based strategy torque curves, cause high consumption peaks during mode switching. The energy consumption results during hybrid operation phases are also better with the proposed strategy thanks to the optimal control algorithm which optimizes hybrid torque split. Otherwise, during pure electric or hydraulic operation of the powertrain, the consumption levels observed with the two control algorithms are relatively similar. From the Table IV, it can be seen that the proposed on-line control algorithm allows to obtain a total gain of consumption of about 8.7% and 7.6% compared to the deterministic rules-based algorithm respectively on UDC and FTP-75 driving cycles. The global minimum energy consumptions obtained during off-line simulation of the proposed control strategy on UDC and FTP-75 driving cycles, are in the two cases only around 2% less than the energy consumption obtained from on-line simulation with uncertainties introduced on the driving condition. More precisely, it corresponds to 2.1% less energy consumption on UDC driving cycle and 1.6% less energy consumption on FTP-75 driving cycle.

TABLE IV
CONSUMPTION RESULTS COMPARISON OF THE DETERMINISTIC RULES-BASED STRATEGY AND THE PROPOSED STRATEGY.

		Rules-based strategy	**Proposed on-line control strategy**	**Off-line control strategy**
UDC	Average instantaneous electricity consumption	6.14 kW	5.71 kW	5.63 kW
	Average instantaneous fuel consumption	13.89 kW (11.10 L/100km)	12.57 kW (10.05 L/100km)	12.28 kW (9.81 L/100km)
	Average total consumption	1.0015 kWh	0.9145 kWh	0.8955 kWh
FTP-75	Average instantaneous electricity consumption	11.82 kW	11.23 kW	11.14 kW
	Average instantaneous fuel consumption	15.43 kW (12.29 L/100km)	13.94 kW (11.08 L/100km)	13.61 kW (10.84 L/100km)
	Average total consumption	12.3679 kWh	11.4231 kWh	11.2384 kWh



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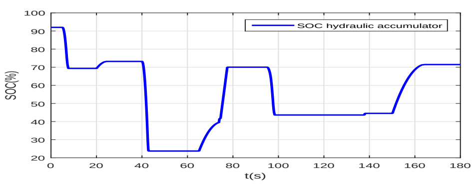

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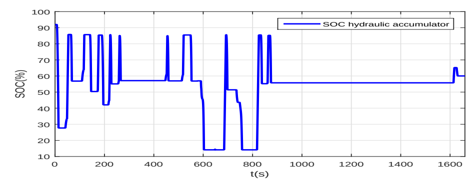
Fig. 16. Hydraulic accumulator state of charge evolution on UDC driving cycle: (a) Deterministic rules-based EMS, (b) Proposed EMS.



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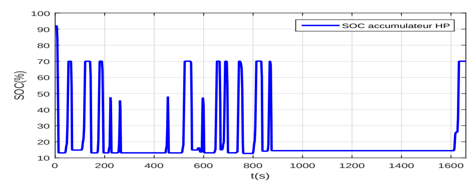

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Fig. 18. Hydraulic accumulator state of charge evolution on FTP-75 driving cycle: (a) Deterministic rules-based EMS, (b) Proposed EMS.

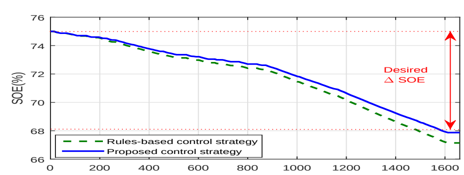

Fig. 17. Battery state of charge evolution on FTP-75 driving cycle.

V. CONCLUSION

This paper focused on the development of a hybrid control architecture for an optimized energy management of a tri-hybrid powertrain of a heavy vehicle. The proposed overall control scheme consists of two control layers and it is based on fuzzy logic and optimal control combination in order to make the most of the benefits of each approach to lead toward a well adapted global control strategy that can be used to deal with the complex characteristics of the studied powertrain. Indeed, the proposed strategy allows to select in real time the most suitable operating mode, optimize energy consumption and avoid the jolts of the motors torque that cause driving inconvenience and premature mechanical fatigue of the powertrain. In addition, through the use of fuzzy logic and adaptive optimal control, robustness and adaptability are also among the main strengths of the proposed control strategy. These two important features allow in particular to deal with the inherent uncertainties in this highly complex system. In particular, driving conditions uncertainties are well taken into account. Validation tests carried out using high-fidelity TruckMaker software have confirmed the effectiveness and validity of this novel approach. The implementation work on the actual bus is in progress and further experimental tests are planned in near future.

VI. ACKNOWLEDGMENT

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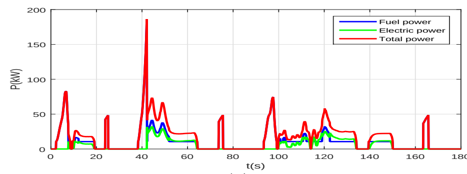

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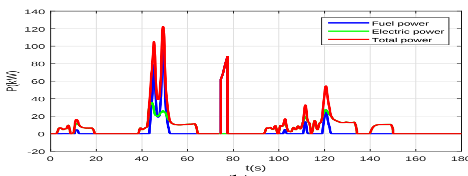

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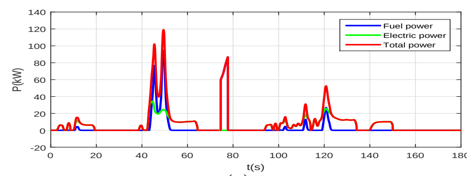

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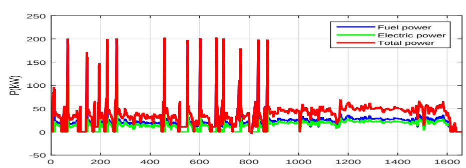
Fig. 19. Energy consumption results on UDC driving cycle: (a) Deterministic rules-based strategy, (b) On-line EMS, (c) Off-line EMS.



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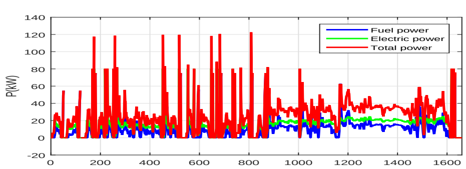

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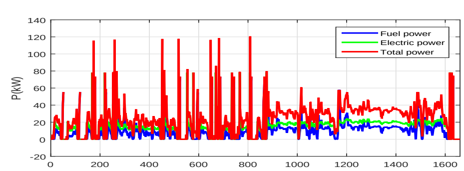

(a)

Fig. 20. Energy consumption results on FTP-75 driving cycle: (a) Deterministic rules-based strategy, (b) On-line EMS, (c) Off-line EMS.

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