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FROM OFFLINE TO ADAPTIVE ONLINE ENERGY MANAGEMENT STRATEGY OF HYBRID VEHICLE USING PONTRYAGIN'S MINIMUM PRINCIPLE

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ABSTRACT−This paper details the development of an energy management strategy (EMS) for real-time control of a multi hybrid plug-in electric bus. The energy management problem has been formulated as an optimal control problem in order to minimize the fuel consumption of the bus drivetrain for a typical day of operation. Considering the physical characteristics of the studied hybrid electric bus and its well-known daily tour, the Pontryagin's minimum principle (PMP) is firstly used as the mean to obtain offline optimal EMS. Afterward, in order to adapt the proposed strategy for real-time implementation, the proposed control parameters are adapted online using feedback from the battery state of energy (SOE) which allows us to accurately control the battery SOE in the presence of wide range of uncertainties. The work proposed in this paper is conducted on a dedicated high-fidelity dynamical model of the hybrid bus, that was developed on MATLAB/TruckMaker software. The performance evaluation of the proposed strategy is carried out using a normalized driving cycles to represent different driving scenarios. Obtained results show that among the investigated methods, it is reasonable to conclude that the proposed adaptive online strategy based on PMP is the most suitable to design the targeted EMS.

KEY WORDS : Optimal control, Heavy hybrid vehicle, Energy management, Pontryagin's minimum principle

SUBSCRIPTS

A : bus frontal area
C_d : drag coefficient
D_{HM} : displacement of the hydraulic motor
D_{HP} : displacement of the hydraulic pump
EM : electric motor
E_{max} : maximum energy stored in the battery
F_{ad} : aerodynamic force
F_g : gravity force
F_{rr} : rolling resistance
F_t : tractive force
g : gravity acceleration
H : Hamiltonian function
HM : hydraulic motor
HP : hydraulic pump
ICE : internal combustion engine
m : mass of the bus
$\dot{m}_f$: fuel flow rate
P_{BAT} : power delivered by the battery
P_{EM} : power consumed by the electric motor
P_F : instantaneous power of the fuel
Q_{LHV} : lower heating value of the fuel
SLR : static loaded radius of the wheel
SOC : battery state of charge
SOE : battery state of energy
T_{HM} : torque of the hydraulic motor
T_{ICE} : torque of the engine
T_{wheel} : torque of the wheel
U : admissible control set
v : velocity of the bus
a : acceleration of the bus
γ, σ : lagrange multipliers used to introduce constraints
$\eta_{m_{HM}}$: mechanical efficiency of the hydraulic motor
$\eta_{m_{HP}}$: mechanical efficiency of the hydraulic pump
$\eta_{v_{HM}}$: volumetric efficiency of the hydraulic motor
$\eta_{v_{HP}}$: volumetric efficiency of the hydraulic pump
η_{BAT} : efficiency of the battery
θ : slope of the road
λ : costate
λ_0 : initial values of the costate
λ_{max} : maximum values of the costate
λ_{min} : minimum values of the costate
μ_{rr} : rolling resistance coefficient
ξ : maximum hydraulic torque variation rate
ρ : density of the air
ρ_1, ρ_2 : gearbox' reduction ratios
ω_{HM} : rotational speed of the hydraulic motor
ω_{ICE} : rotational speed of the engine
ω_{wheel} : rotational speed of the wheel

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1. INTRODUCTION

Electric vehicles have emerged, at first, as the most promising solution to address the pollution problems which are increasingly critical. But several decades later, still victim of their low autonomy and the excessive cost of their batteries, they struggle to compete with conventional internal combustion engine based vehicles. A mid-term solution seems to emerge over the last decade: Hybrid electric vehicles (HEV). A hybrid vehicle uses, by definition, at least two energy sources to ensure its propulsion. Generally, at least two motors are associated with the mechanical transmission elements to ensure the traction of the vehicle. The arrangement of these elements defines the vehicle architecture. There are many possible drivetrain architectures such as series hybrid, parallel hybrid or power-split hybrid drivetrain configurations (Chan *et al*., 2010). The advantage of the hybridization of drivetrains is to overcome the two main drawbacks of internal combustion engines that are the low energy efficiency and the power irreversibility which makes the engine unable to retrieve the energy incurred during braking. Hybridization will therefore draw on the strengths of different types of engines by combining the excellent efficiency and reversibility of electric motors with the high energy density of fossil fuels which guarantees the autonomy, limits the vehicle weight and reduces refueling time.

One way to reduce fuel consumption and pollutant emissions of HEV is, as for conventional vehicles, improvement of various mechanical parts of the vehicle (lightening of vehicle body, improving aerodynamic performance, using new technology for the engine, etc.). The presence of additional power sources in the HEV introduces additional degrees of freedom in controlling the drivetrain, since at each time the driver's power request can be delivered by either one of the on-board energy sources or their combination. The additional degrees of freedom can be leveraged to reduce fuel consumption and pollutant emissions and also to optimize other possible cost such as battery life (Shen and Khaligh, 2015). However, controlling HEV raises new problems to find the most efficient way of deciding the power distribution between the power sources. This task is performed by the energy management strategy which is the highest control layer of the drivetrain's Control strategy (Serrao *et al*., 2013).

In commercially available HEV, the energy management has been traditionally performed using heuristic controllers in which rules are designed to manage the on-board energy of the vehicle (Kamal *et al*., 2017; Wu *et al*., 2012). Such control strategies are effective in real-time implementation but they require a careful calibration of the parameters (Boukehili *et al*., 2012). A significant improvement with respect to such strategies is achieved with model based optimal control methods. These methods can be divided into numerical and analytical approaches. In numerical optimization methods like dynamic programming (Abdrakhmanov and Adouane, 2017; Dinmen and Gven, 2012; Ximing *et al*., 2015), the global optimum is found numerically under the assumption of full knowledge of the future driving conditions. Unfortunately, the results obtained through dynamic programming cannot be implemented directly due to its high computational demands. To overcome this problem, approximated dynamic programming (Johannesson *et al*., 2007) and stochastic dynamic programming (Johannesson *et al*., 2007; Moura *et al*., 2011) had been proposed as solutions. Analytical optimization methods, on the other hand, use a mathematical problem formulation to find an analytical solution that makes the numerical solution faster than the purely numerical methods. Within this category, Pontryagin's minimum principle based energy management strategy is introduced as an optimal control solution (Tang *et al*., 2015; Teng *et al*., 2014; Yan *et al*., 2014). This approach can only generate an optimal solution if implemented offline. For online implementation Equivalent Fuel Consumption Minimization (ECMS) methods that lead to suboptimal solutions have been proposed for HEVs (Cai *et al*., 2017). ECMS is based on instantaneous optimization. Although, it is suitable to be implemented in real-time. Model predictive control based methods have been also applied to solve online the energy management problem (Fengjun *et al*., 2012). One of the main drawbacks of this approach is the high computational power required to calculate the optimal power split at each sampling interval.

This paper details the development of energy management strategies to optimize the power distribution in a plug-in hybrid bus actuated by three distinct types of power (internal combustion engine, electric motor and hydraulic motor). Among the energy management strategies discussed above, Pontryagin's minimum principle based optimization turns out to be the most appropriate approach to design an energy management strategy for the considered hybrid bus since it can guarantee, under given conditions, near optimality while keeping the methodology simple (Kim and Rousseau, 2012). Furthermore, since the route of the bus, roads levels variations and even traffic lights are well known, prediction of optimal velocity trajectory for the trip can be carried out (Wu *et al*., 2014; Zheng *et al*., 2016). This available information can be exploited to make the bus more efficient and to ensure the desired battery depleting level. Few authors seem to have explored this path to go in designing energy management strategies, whose use will affect a small part of passenger vehicles. Thus, in this work, an adaptation of Pontryagin's minimum principle based energy management strategy to a plug-in multi hybrid bus is proposed and the available information on optimal velocity trajectory are exploited in order to achieve the most efficient way of operation in the studied hybrid bus application while ensuring the desired battery depleting level. The key contributions are firstly in formulating the optimization problem so as all the sources of power of the studied hybrid bus are considered by the optimization algorithm. Secondly, the general concepts initially presented in literature are improved by taking into account the motors dynamic limits. And finally, the control parameters are tuned online permitting to take into account the traffic conditions change (making therefore the proposed adaptive strategy suitable for online implementation). The proposed adaptive Pontryagin's minimum principle based energy management strategies have been also evaluated with regards to the existing rules-based energy management strategy to assess performances of the proposed strategy.

The paper is structured as follows: Section 2 describes the studied hybrid bus architecture and model. Section 3 introduces the proposed energy management strategy. In Section 4, several simulations results are presented showing the efficiency of the proposed energy management strategies. Finally, conclusions and some prospects are given in the last section.

2. MODELING OF THE HYBRID BUS

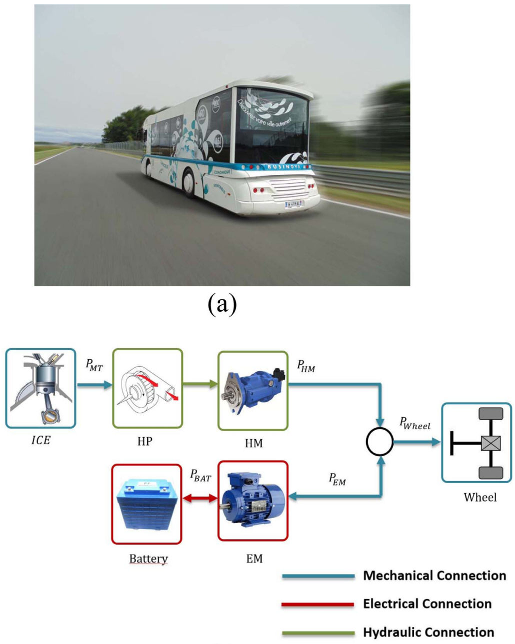
The aim of this section is to illustrate the architecture and the mathematical model of the studied system, i.e., BUSINOVA hybrid bus, developed by SAFRA (2017) (cf. Figure 1 (a)). This bus is composed of an electric motor, a hydraulic motor, an internal combustion engine and battery as the propulsion drivetrain system of the vehicle. The electric motor is a 103 kW permanent magnet electrical machine from Visedo® developed especially for heavy duty applications. It has six pole pairs and its nominal voltage is 500 V (VISEDO®, 2014). The internal combustion engine is produced by VM Motori®. It delivers a maximum torque of 340 N.m at 1400 rpm and its maximum produced power is 70 kW (VM Motori®, 2015). The hydraulic motor is a Parker® V14 series with a displacement that varies between 22 and 110 cm^3 (Parker®, 2014).

2.1. Hybrid Bus Drivetrain Architecture

The model of the studied hybrid bus is based on a series-parallel power-split hybrid architecture. A simple block diagram of the power flows on the bus is shown in Figure 1 (b).

The electric and hydraulic motors are both directly connected to the transmission and can ensure simultaneously or independently the traction of the bus. On the other hand, the internal combustion engine is coupled to a hydraulic pump for driving the hydraulic motor and therefore allowing the engine load shifting. This will permit the engine to be run under a more efficient operating range (Changwei *et al.*, 2013).

The rotational speeds of the hydraulic motor and the electric motor are imposed by the wheels speed in proportion to the reduction ratios of hydraulic and electric motors respectively. Moreover, the rotational speed ω_{HM} and the torque T_{HM} of the hydraulic motor are expressed as a function of the rotational speed and the torque of the internal combustion engine as follows.



(b)

Figure 1. Businova hybrid bus: (a) Photography of the bus; (b) Block diagram of the bus' drivetrain power flows (ICE: Internal Combustion Engine, HP: Hydraulic Pump, HM: Hydraulic Motor, EM: Electric Motor).

$$\begin{cases} \omega_{HM}(T_{ICE}, D_{HM}) = \dfrac{D_{HP} \cdot \eta_{v_{HM}} \cdot \omega_{ICE}}{D_{HM} \cdot \eta_{v_{HP}}} & \text{(1a)} \\[2ex] T_{HM}(T_{ICE}, D_{HM}) = \dfrac{D_{HM} \cdot \eta_{m_{HM}} \cdot T_{ICE}}{D_{HP} \cdot \eta_{m_{HP}}} & \text{(1b)} \end{cases}$$

where ω_{ICE}, T_{ICE} are respectively rotational speed and torque of the engine, and D_{HM}, D_{HP}, $\eta_{m_{HM}}$, $\eta_{m_{HP}}$, $\eta_{v_{HM}}$, $\eta_{v_{HP}}$ are respectively the displacements, mechanical efficiency and volumetric efficiency of the hydraulic motor (HM) and the hydraulic pump (HP).

2.2. Dynamical Model

The first step in the modeling of our system is to produce the equations of the bus dynamics. The purpose of the dynamic model is to have a realistic global behavior of the bus in order to test the proposed optimization techniques. To describe a generic case, let us assume that the bus is moving up the slope of θ degree (cf. Figure 2). The origin of the coordinates is situated in the center of mass (CoM). We suppose that CoM of the bus is in its geometric center. Projecting the vectors of the forces to x-axis (the bus is moving along x-axis in the positive direction, with the velocity v and acceleration $a_x = a$), we obtain the following expressions of the forces acting on the bus:

$$F_t - F_{rr} - F_{ad} - F_g \sin(\theta) = m\, a \quad (2)$$

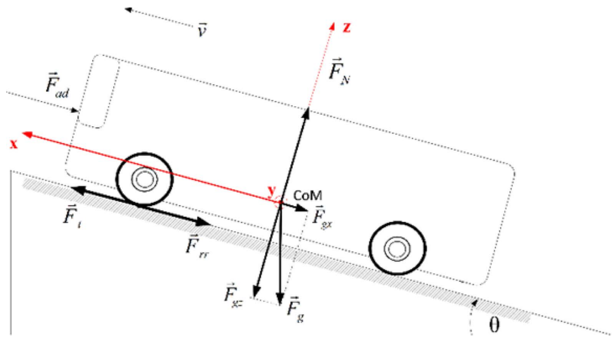


Figure 2. Forces acting on the bus.

where F_t the tractive force, F_{rr} the rolling resistance, F_{ad} aerodynamic force, F_g the gravity force, m the mass of the bus, and g the gravity acceleration.

Equation (2) can be rewritten as follows:

$$a = \frac{dv}{dt} = \frac{F_t - F_{rr} - F_{ad} - F_g \sin(\theta)}{m} \tag{3}$$

• The rolling resistance F_{rr} appears mainly due to the friction of the vehicle tires on the road. It is proportional to the vehicle weight (Larminie and Lowry, 2003).

$$F_{rr} = \mu_{rr} m \cdot g \cdot \cos(\theta) \tag{4}$$

where μ_{rr} is rolling resistance coefficient.

• Aerodynamic force F_{ad} is the part of the force due to the friction of the vehicle body moving through the air. It is calculated as (Larminie and Lowry, 2003):

$$F_{ad} = \frac{1}{2} \rho A C_d v^2 \tag{5}$$

where ρ is the density of the air, A is the bus frontal area and v is the bus velocity. C_d constant called drag coefficient that depends on the shape of the vehicle.

Otherwise, the wheel torque could be described by the product of all the forces that oppose the movement of the bus by the static loaded radius (*SLR*) of the wheel which is the radius from the wheel center down to the ground, at reference load and pressure. The *SLR* is considered identical to all the wheels and constant over time.

$$T_{wheel} = (F_{rr} + F_{ad} + F_g \sin(\theta)) SLR \tag{6}$$

2.3. Control Oriented Model

The amount of residual energy of the battery traditionally represented by the estimation of the battery state of charge *SOC* (Tang *et al*., 2015) or the battery state of energy *SOE* (Mura *et al*., 2015) is the main dynamic state in optimal control of HEVs (Roy *et al*., 2014; Stockar *et al*., 2011). In particular, the state equation connects the variation of the battery's remaining energy to the control variable of the system. In the formulation of the energy management problem of the hybrid bus studied in this paper, the *SOE* instead of the *SOC*, is considered as the dynamic state $x(t)$. There are several advantages of using the estimated *SOE* to represent the battery residual energy. Indeed, the energy loss on the internal resistance, the electrochemical reactions and the decrease of the battery voltage are considered in the *SOE* estimation (Liu *et al*., 2014). Based on the previous assumption of using estimated *SOE* to represent the battery residual energy, the control oriented model can be represented by:

$$\dot{x}(t) = f(x(t), u(t), w(t)) \tag{7}$$

where

$$x(t) = SOE(t),\ u(t) = \begin{bmatrix} T_{HM} \\ \omega_{HM} \end{bmatrix},\ w(t) = \begin{bmatrix} T_{wheel} \\ \omega_{wheel} \end{bmatrix} \tag{8}$$

$u(t)$ is the control input and $w(t)$ is an exogenous input. The above model can be rewritten as follows.

$$\dot{x}(t) = \frac{d\,SOE(t)}{dt} = -\frac{P_{BAT}}{E_{max}} = -\frac{P_{EM}}{\eta E_{max}} \tag{9}$$

Depending on whether the battery is in discharging phase ($\frac{d\,\mathrm{SOE}}{dt} \leq 0$) or in charging phase ($\frac{d\,\mathrm{SOE}}{dt} \geq 0$), η is defined as follows (Tremblay *et al*., 2007).

$$\eta = \begin{cases} \eta_{BAT} & \text{in discharging phase} \\ 1/\eta_{BAT} & \text{in charging phase} \end{cases} \tag{10}$$

Equation (9) is obtained from the battery internal resistance model (Tremblay *et al*., 2007). In this equation, E_{max} is the maximum energy that can be stored in the battery, η_{BAT} is the efficiency of the battery, P_{BAT} is the power delivered by the battery and P_{EM} is the power consumed by the electric motor to produce torque T_{EM} at speed ω_{EM}.

3. ENERGY MANAGEMENT STRATEGY

The following section details the design procedure of the proposed energy management strategy. Its aim is to find the optimal power split between the different power sources. The optimal control problem formulation is firstly presented and then the analytical expression of the proposed solutions, for offline and after for online optimization, are described. These power management strategies are based on the findings of the Pontryagin's minimum principle.

3.1. Optimal Control Problem Formulation

The objective of the energy management strategy proposed in this paper is to decide how to split the driver's demanded power between the different power sources of the hybrid drivetrain to optimize the selected criterion without sacrificing the bus drivability. Since our primary goal is to minimize the energy consumption of the bus, the energy management problem is formulated as an optimal control problem. The objective is to find, at each sample time, the optimal value of the control input that minimizes a cost function representing the power consumption of the drivetrain. This minimization of the cost function must be done under a certain number of constraints. In fact, the drivetrain components dimensioning imposes minimum and maximum limits on the exchanged powers. These limits form the following constraints.

The internal combustion engine and electric motor have limited operating ranges. Therefore, provided or absorbed torques must be comprised between minimum and maximum limits.

$$T_{EM}^{min} \leq T_{EM}(t) \leq T_{EM}^{max} \tag{11}$$

$$T_{HM}^{min}(T_{ICE}^{min}, D_{HM}) \leq T_{HM} \leq T_{HM}^{max}(T_{ICE}^{max}, D_{HM}) \tag{12}$$

The maximum and minimum torque limits of the internal combustion engine and electric motor varies according to the variation of the system's operating point (torque-speed). Look-up tables are therefore used to determine their values at each time.

The instantaneous power demand of the drivetrain should always be satisfied, which results in,

$$\rho_1 T_{HM}(T_{ICE}, D_{HM}) + \rho_2 T_{EM}(t) + T_{wheel}(t) = 0 \tag{13}$$

where ρ_1 and ρ_2 are the gearbox' reduction ratios of hydraulic and electric motors respectively.

Compared with energy management problem formulation for charge sustaining HEV (Mura *et al*., 2015; Stockar *et al*., 2011; Tang *et al*., 2014), there is no constraint on the final *SOE* for plug-in HEV allowing the charge depleting operation. Thus, the energy consumed on the entire cycle does not come exclusively from the fuel since most of the available electrical energy is supplied from the grid. This implies that the cost function must take into account all the energy sources used to ensure the traction of the bus.

In this paper, the cost function J to be minimized over the time interval $[t_i, t_f]$ is defined based on the total electric and fuel energy consumed by the vehicle as follows.

$$J = \int_{ti}^{tf} a.P_F(u(t)) + (a-1).P_{BAT}(u(t))dt \tag{14}$$

where $a \in [0,1]$ is a weighting coefficient used to make a balance between the two sub-criterion, P_F is the instantaneous power of the fuel (engine power input). As in several other papers dealing with this topic (Mura *et al*., 2015; Rousseau *et al*., 2007), P_F is expressed in terms of the fuel flow rate $\dot{m}_f$ and the lower heating value of the fuel (Q_{LHV} = 43 MJ/kg) using the formulation given in Equation (15).

$$P_F(u(t)) = \dot{m}_f(u(t))\ Q_{LHV} \tag{15}$$

The control variables (T_{HM} and ω_{HM}) are linked together through the hydraulic motor dynamics, therefore, there can only be one target control value at a time. In this paper, we have chosen to leave the rotation speed free so that it will be imposed by the wheels speed. The hydraulic motor torque is thus the only remaining control variable that can be used to decide how to split the driver's demanded power.

The optimization problem is then to find the hydraulic torque that should be provided at every sample time in order to minimize the total energy consumed while checking the constraints thus mentioned above (cf. Equations (11) to (13)). To these constraints it is added a new constraint (16) which aims to limit the admissible control region in order to take into account the limits of the hydraulic motor dynamics and consequently taking into account the limits of the internal combustion engine dynamics.

$$\frac{dT_{HM}}{dt} - \xi \geq 0 \tag{16}$$

with ξ is the maximum hydraulic torque variation measured over a short period of time.

To introduce constraints in the optimization problem, these are transformed into equality constraints. The constraint (16) can be rewritten as follows (Wang and Wah, 1998).

$$\frac{dT_{HM}}{dt} - \xi - \dot{\varepsilon}^2 = 0 \tag{17}$$

where ε is a slack variable.

By using Equation (13), it is possible to rewrite the constraints (11) and (12) as a single constraint on the control variable as follows.

$$\tilde{T}_{HM}^{min}(T_{HM}^{min}, T_{EM}^{max}) \leq T_{HM} \leq \tilde{T}_{HM}^{max}(T_{HM}^{max}, T_{EM}^{min}) \tag{18}$$

with

$$\tilde{T}_{HM}^{min} = \max(\rho_1 \cdot T_{HM}^{min}, T_{wheel} - \rho_2 . T_{EM}^{max}) \tag{19}$$

$$\tilde{T}_{HM}^{max} = \max(\rho_1 \cdot T_{HM}^{max}, T_{wheel} - \rho_2 . T_{EM}^{min}) \tag{20}$$

It means that when the torque applied to the wheel is too significant to be only produced by the electric motor, the $\tilde{T}_{HM}^{min}$ limit imposes a minimum torque on the hydraulic motor. Additionally, $\tilde{T}_{HM}^{max}$ limit prevents the electric motor torque set-point to become less than T_{EM}^{min}.

Finally, using a 2nd order approximation (Rousseau *et al*., 2007), the constraint (18) is written as the equivalent form given by (21),

$$-T_{HM}^2 + \alpha T_{HM} + \beta = 0 \tag{21}$$

with

$$\alpha = \tilde{T}_{HM}^{max} - \tilde{T}_{HM}^{min} \tag{22}$$

$$\beta = \tilde{T}_{HM}^{max} \cdot \tilde{T}_{HM}^{min} \tag{23}$$

3.2. Offline Energy Management Control Algorithm

With the optimization problem fully defined, Pontryagin's minimum principle can be used to give numerical solution.

According to Pontryagin's minimum principle, minimizing the cost function given in (14) is equivalent to minimizing the Hamiltonian function H of the system at each instant of time.

$$H(x(t), u(t), \lambda(t)) = a.P_{\mathrm{F}}\left(\rho_1 T_{\mathrm{HM}}(t), \frac{1}{\rho_1}\omega_{\mathrm{HM}}(t)\right)$$
$$-\left(\frac{\lambda(t)}{\eta E_{\max}} - (a-1)\right) P_{\mathrm{ME}}\left(\rho_2 T_{\mathrm{EM}}(t), \frac{1}{\rho_2}\omega_{\mathrm{EM}}(t)\right) \tag{24}$$

where $\lambda(t)$ is the costate (or the Lagrange multiplier). For the considered energy management problem, an extended Hamiltonian function is defined to account for the constraint (17) and (21). The additional terms are introduced using a new Lagrange multipliers (i.e., $\gamma(t)$ et $\sigma(t)$ respectively).

$$H(x(t), u(t), \lambda(t), \gamma(t), \sigma(t)) =$$
$$P_{\mathrm{F}}\left(\rho_1 T_{\mathrm{HM}}(t), \frac{1}{\rho_1}\omega_{\mathrm{HM}}(t)\right) - \left(\frac{\lambda(t)}{\eta E_{\max}} - (a-1)\right)$$
$$P_{\mathrm{ME}}\left(\rho_2 T_{\mathrm{EM}}(t), \frac{1}{\rho_2}\omega_{\mathrm{EM}}(t)\right) + \gamma(t)(-T_{\mathrm{HM}}^2 + \alpha T_{\mathrm{HM}} + \beta)$$
$$+ \sigma(t)\left(\frac{dT_{\mathrm{HM}}}{dt} - \xi\right)^2 \tag{25}$$

Then the necessary conditions of the hybrid drivetrain control problem can be defined via calculus of the derivative of the Hamiltonian function as follows:

$$\frac{\partial H(t)}{\partial u(t)} = \frac{\partial H(t)}{\partial T_{\mathrm{HM}}(t)} = 0 \tag{26}$$

$$-\frac{\partial H(t)}{\partial x(t)} = \frac{\partial H(t)}{\partial SOE(t)} = \dot{\lambda}^*(t) \tag{27}$$

$$\frac{\partial H(t)}{\partial \lambda(t)} = \dot{x}^*(t) \tag{28}$$

$$\frac{\partial H(t)}{\partial \gamma(t)} = -T_{\mathrm{HM}}^2 + \alpha T_{\mathrm{HM}} + \beta = 0 \tag{29}$$

$$\frac{\partial H(t)}{\partial \sigma(t)} = \left(\frac{dT_{\mathrm{HM}}}{dt} - \xi\right)^2 = \dot{\varepsilon} \tag{30}$$

The costate λ is determined by the condition (28).

While taking into account the battery discharge characteristics shown in Figure 3, it is clear that battery open circuit voltage is relatively independent on the battery state of energy SOE and thus the power consumed by the electric motor $P_{\mathrm{EM}}(t)$ is also independent on the battery SOE (Kirk, 2012; Lino and Sciarretta, 2007). Under this assumption, it is straightforward to consider that the costate λ is a constant value during the entire driving cycle since the derivative of the Hamiltonian function H in (28) is null in this case.

The condition (26) determines the optimal control trajectory $T_{\mathrm{HM}}^*(t)$. If this necessary condition is satisfied, then the optimal hydraulic torque $T_{\mathrm{HM}}^*(t)$ must be given by Equation (31).

$$T_{\mathrm{HM}}^*(t) = \arg \min_{T_{\mathrm{HM}} \in U} H(SOE(t), T_{\mathrm{HM}}(t), \lambda(t)) \tag{31}$$

where U is defined as the admissible control set.

After the hydraulic motor torque is obtained, the internal combustion engine torque and speed are calculated depending on the desired speed and torque of the hydraulic motor. Thanks to the displacement tuning capability of the hydraulic motor, the internal combustion engine load can be shifted freely to operate this latter one close to its maximum efficiency curve. Especially in this case, the speed of the internal combustion engine is not imposed by the wheels speed and it can be set to a nearly constant value where the engine is the most efficient. To reach this goal the displacement of the hydraulic motor is controlled online by using Equation (1a). Thereafter, the engine torque is calculated as a function of the displacement and the optimal torque of the hydraulic motor by using Equation (1b).

The prior knowledge of the future driving condition is used to search iteratively the value of the costate λ that generates the correct final SOE_f at the end of the daily duty time of the bus. In fact, for a given driving cycle there exists only one value of the costate for which the solution that minimizes the Hamiltonian H at each sample time is also the one that satisfies the terminal condition on the final value of SOE. This corresponds to the global optimal solution of the problem. In this paper, it is considered that the desired final value of SOE after eight hours of driving is 17 %. The working hypothesis behind this assumption is to use the maximum amount of energy that can be consumed from the battery in one day driving.

3.3. Online Sub-optimal Energy Management under Uncertainty

Using available information about the driving cycle of the bus (road profile, the different bus stops, rough approximation of the road trafic, etc.), the global optimization problem can be solved offline in order to have the optimal power distribution between the power sources. However, in practice this available information is usually

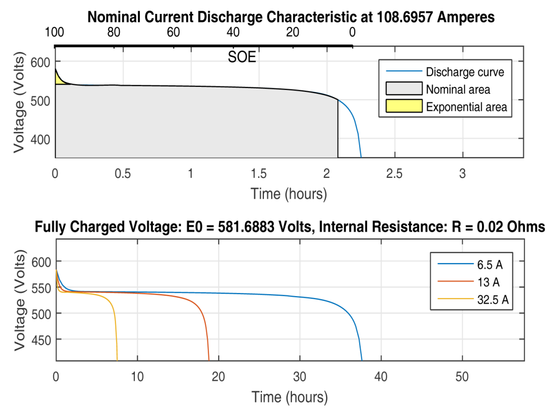


Figure 3. Discharge characteristics of the used battery.

uncertain because of the variation of traffic conditions. Thus, if uncertainties on the driving cycle are considered, it will not be possible to find an exact value of the costate which satisfies the terminal condition on the final *SOE*. This issue can be overcome with the use of online optimization, however, only sub-optimal behavior is achievable in this case. Since the optimal control algorithm is independent of the driving condition, the value of the costate is the only parameter that should be tuned while calculating the online sub-optimal power split. The online optimization algorithm proposed in this paper adapts the costate in real-time in order to achive the desired final *SOE* value at the end of the considered driving interval. To reach this goal, the optimal state trajectory calculated by offline optimization is used online to guide the choice of the costate value. The objective here is not to track the offline *SOE* trajectory but to use the information about the driving cycle that it contains (acceleration, braking, road slope, etc.) to adapt the costate value depending on the characteristics of the route and the new driving conditions. As shown in Figure 4, *SOE* information is fed back at each sample time to operate the online optimization algorithm in closed loop. This will ensure that the *SOE* will always reach its desired final value despite the lack of knowledge of the future driving conditions.

The value of the costate is found at each sample time according to (32).

$$\lambda(t) = \tau(t)\lambda_{\max} + (1-\tau(t))\lambda_{\min} \tag{32}$$

with

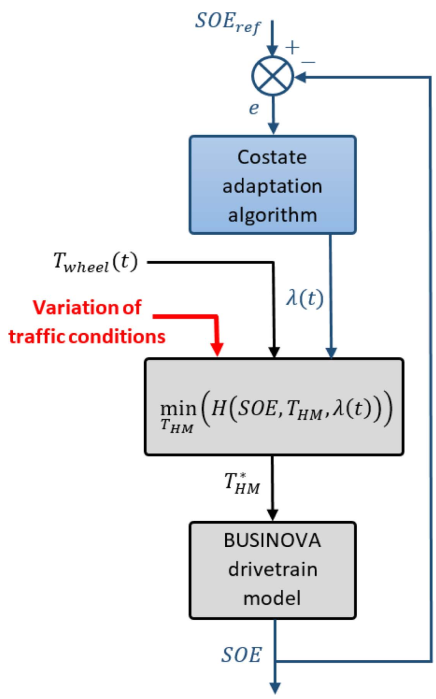


Figure 4. Block diagram of the proposed adaptive online power management strategy.

$$\lambda(t)|_{\tau_0=0.5} = \lambda_0 = \frac{\lambda_{\min}+\lambda_{\max}}{2} \tag{33}$$

λ_0, $\lambda_{\min}$ and $\lambda_{\max}$ are respectively the initial, the minimum and the maximum values of the costate λ. The costate variation range (i.e., $[\lambda_{\min}, \lambda_{\max}]$) is chosen sufficiently large to handle all types of uncertainties on the knowledge of the driving cycle including unplanned stops. $\tau(t)$ is a tuning parameter such as $\tau \in [0,1]$ and τ_0 is its initial value fixed at 0.5. The problem of the evaluation of $\lambda(t)$ is therefore transferred to the evaluation of $\tau(t)$.

The parameter $\tau(t)$ is estimated in real-time using the *SOE* feedback as stated in Equation (34).

$$\tau(t) = \tau_0 - \frac{\mu(SOE(t)-SOE_{\text{ref}}(t))}{\Delta SOE_{\max}} \tag{34}$$

where SOE_{ref} is the optimal *SOE* trajectory calculated offline, $\Delta SOE_{\max}$ is the maximum amount of energy that can be consumed from the battery during the whole drive cycle, and μ is a constant calibration parameter.

The objective of the suggested formula in Equation (34) is to find at each time the value of τ needed to bring back the actual *SOE* to its desired value SOE_{ref}. In other words, when the battery *SOE* has a different value from the desired SOE_{ref}, the parameter τ is modified to give priority to the use of the electric motor or to the hydraulic motor and thus it tries to discharge the battery or, on the contrary, to capture as much braking energy as possible to charge the battery.

The strength of the proposed method is its ease of design and configuration. It contains only a single calibration parameter which can be obtained experimentally. Furthermore, unlike model predictive control based methods where the future driving cycle is predicted involving high computational power requirements (Fengjun *et al*., 2012), this method offers accurate tuning of the costate (cf. Section 4) while being very simple to implement thanks to its high computational efficiency.

4. RESULTS AND ANALYSIS

In this section, performance evaluation of the proposed energy management strategies is, firstly, carried out using a normalized driving cycles which represent different usage conditions of a hybrid electric bus including urban and extra-urban driving environments. A further test is then conducted in order to validate the proposed strategies under the real operating conditions of the bus.

4.1. Performance Evaluation of the Proposed EMS

In order to apply the Pontryagin's Minimum principle based energy management strategy for calculating the optimal offline solution, the costate of the energy management strategy has been obtained according to the characteristics of the considered driving cycle (cf. Figure 6 (a)). To recall, the value of the costate is fixed in order to deplete the *SOE* of the battery from its initial state to its desired final state during a full day driving period (cf. Section 3.3). On the other hand, the costate of the adaptive energy management strategy is tuned online based on the predefined driving cycle and to the observed traffic conditions.

To resolve the formulated optimal control strategy, the partial differential terms given in Equation (26) are calculated using the efficiency characteristics data of the electric motor and the hydraulic motor coupled to the internal combustion engine (cf. Figure 5). This data is implemented in Matlab/Simulink as look-up tables. It should be also noted that the regenerative braking is controlled separately during deceleration phases. The braking torque split is calculated by a regenerative braking strategy which allows recovering the maximum amount of vehicle kinetic energy as long as the regenerative torque is smaller than a threshold value defined depending on the actual bus speed (220 N.m for low speed and up to 600 N.m for high speed) to satisfy the drivability constraint. Otherwise, the mechanical braking system adds the braking power needed to meet the driver demands.

Figure 6 shows the simulation results of the offline energy management strategy over the considered driving cycle. A zero road slope is assumed in order to perform this test. The contribution of the fuel energy and the electric energy to the total torque at the wheels is illustrated in Figure 6 (b). The SOE profile is also illustrated in Figure 6 (c). According to Figure 6 (b), it is shown that the distribution of the torque demand between the electric motor and the hydraulic motor is correctly assured and the required torque at the wheels is totally satisfied over the entire driving cycle. Since the information on future driving condition is known in prior, the proposed energy management strategy finds the optimal torque split which operates the engine around its maximum efficiency curve to minimize the power consumption of the drivetrain. The fluctuation range of the toque delivered by the engine is directly related to the amount of electric energy available for electric assist and it allows to always satisfy the constraints of the final SOE of the battery. Otherwise, the energy management strategy doesn't use the engine to charge the battery, because its efficiency is too low and thus recharging the battery using fuel energy is not cost-effective.

It is to be noted that the proposed optimal PMP based algorithm explicitly takes into account the system dynamic

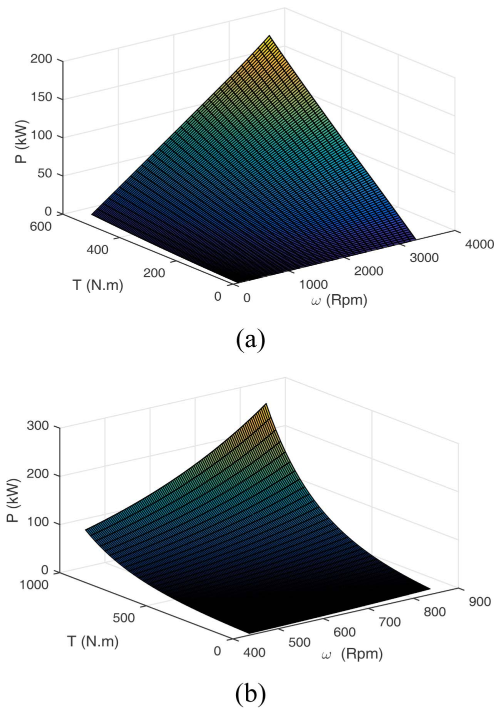


Figure 5. Power consumption mapping: (a) EM; (b) HM coupled to ICE.

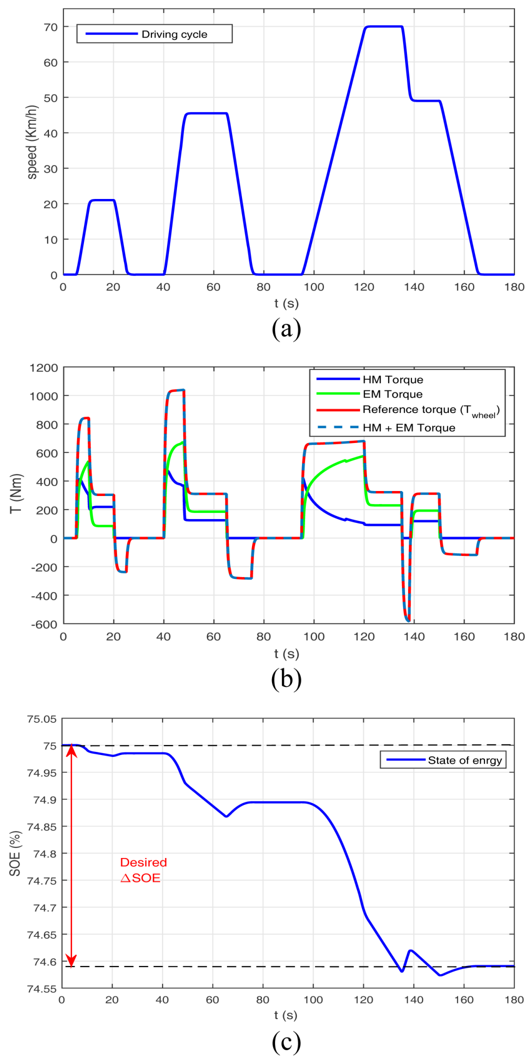


Figure 6. Simulation results of the offline energy management strategy: (a) Driving cycle; (b) Torque distribution profile; (c) *SOE* profile.

through a constraint, which restricts the instantaneous variation rate of the control input. In order to evaluate the optimality of the power split profile obtained by this algorithm, a validation batch test was carried out on the reduced driving cycle shown in Figure 7 (a). In fact, several curves approaching locally the optimal power curve of the hydraulic motor (i.e., by producing local variations of the obtained optimal power curve) have been randomly generated while respecting, on one hand, the constraint on the final *SOE* of the battery and ensuring, on the other hand, that for each new splitting strategy the bus reaches the same final position and velocity (cf. Figure 7 (b)). The electric motor power curves are generated in the ways to provide the additional power amount needed to meet the overall power demand.

The results of this validation test are summarized in Figures 7 (c) and (d). In the bar graph of Figure 7 (c), the energy consumption obtained from the proposed optimal power distribution algorithm is represented by the first bar. The corresponding energy consumption is equal to 22.44 kWh as it is highlighted by the red horizontal line that one can see in this figure. It can be noticed from this figure that the minimum energy consumption is actually achievable by the PMP based energy management strategy.

In Figure 8 (a) an example of a driving cycle obtained under the assumption of unknown traffic conditions is illustrated by a continuous line. It is supposed to represent the effects of fluctuating traffic conditions when the driver tries to follow the regular cycle represented by the dashed line. The total traveled distance is the same as in the regular driving cycle, but the driver behavior is different (i.e., quicker or slower accelerations/decelerations). In addition, an unplanned stop is introduced in this driving cycle to represent situations that induce a high level of uncertainty (i.e., traffic jam, traffic lights, etc.). The amount of uncertainties variation range introduced on this new driving cycle is based on statistical analysis of traffic data collected on the bus route. Thus, uncertainties variation range estimation only takes into account the recurring events that have occurred in the past. As one can observe on Figure 8 (b), lack of knowledge of traffic conditions

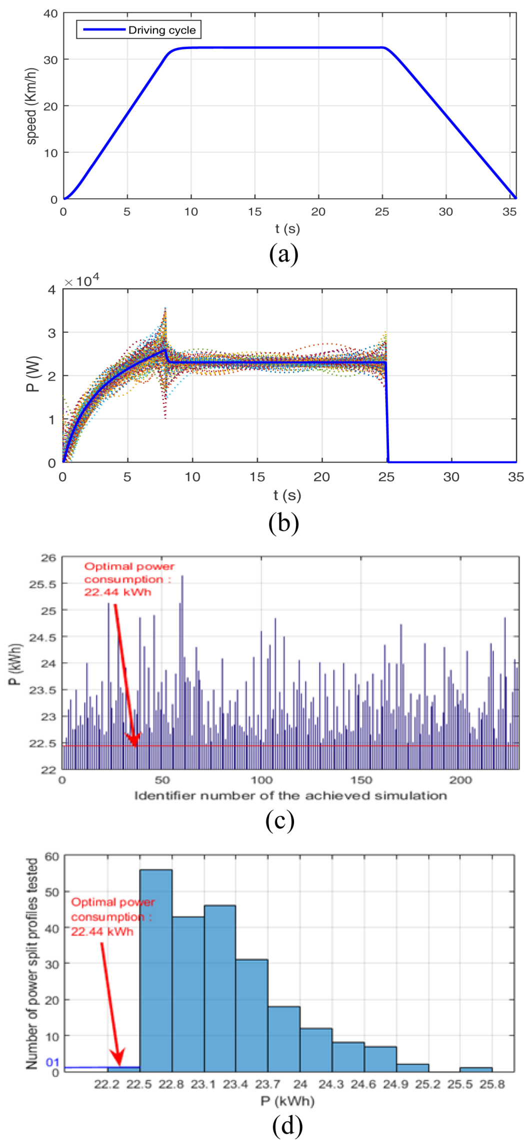


Figure 7. Validation results: (a) Driving cycle; (b) Randomly generated curves (dashed curves) and optimal power profile (continuous curve); (c) Power consumption from the generated test curves vs Optimal power consumption; (d) Distribution of consumed power.

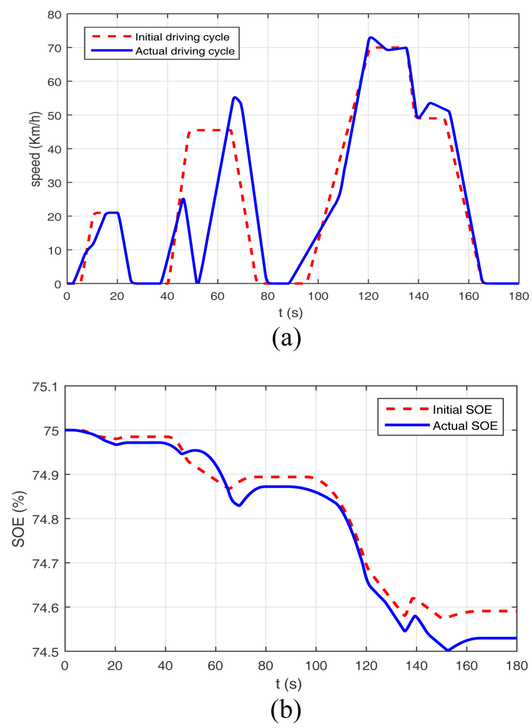


Figure 8. Driving cycle used to simulate unknown (or fluctuating) traffic condition: (a) Driving cycle; (b) *SOE* trajectory from the offline strategy.

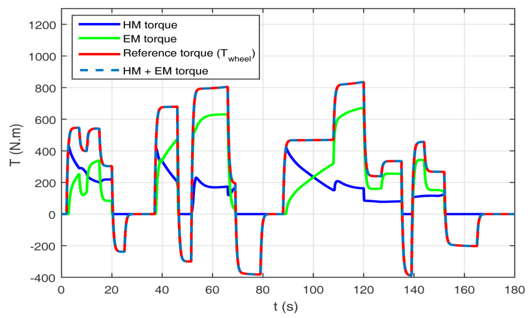
affects the ability of PMP based energy management strategy to respect the constraints on the final *SOE* and consequently, the bus uses more battery energy than the one allowed as can be seen in Figure 8 (b). To cope with this important issue, the proposed adaptive PMP based energy management strategy adapts online the value of the costate λ. The performances of the online energy management strategy are stated in Figure 9. As shown in this Figure, the online energy management strategy offers comparable performances to those obtained by the offline strategy. The *SOE* of the battery stays near to the desired final value and have only a very small difference.

Figure 10 (a) reports a comparison between the power consumption obtained by the two energy management strategies (offline strategy represented by the results set "1" in x-axis and online strategy represented by the results set "2") over the driving cycle given in Figure 8 (a). As shown in this Figure, the power consumption results in both cases is very similar. In fact, the total amount of power consumed in the online strategy case is only increased by 2.8 % compared with the best power economy case obtained by the offline strategy.

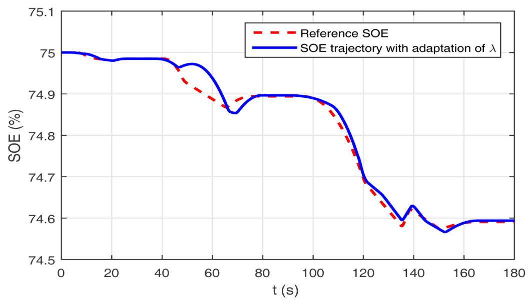
To further assess the effectiveness of the developed online energy management strategy, its power consumption results are compared to the results of a typical rules based energy management strategy (Kamal *et al*., 2017). A rule based strategy is a heuristic energy management method based on the use of a set of deterministic commutation rules to split the total power demand between the motors. In this paper, the rules based strategy used for comparison, was developed based on the power split results from the adaptive PMP based energy management strategy. In fact, these results were analyzed to observe recurring behavior that could be replicated online using deterministic rules.

Figure 11 shows the simulation result of the rules based energy management strategy while the power consumption comparison results are shown in Figure 10 (b). As one can notice from this Figure, the simulation using the proposed adaptive PMP based energy management strategy is considerably more efficient than that of the conventional rules based energy management strategy. A total power economy of about 12.4 % is obtained in this case while using the adaptive PMP strategy.

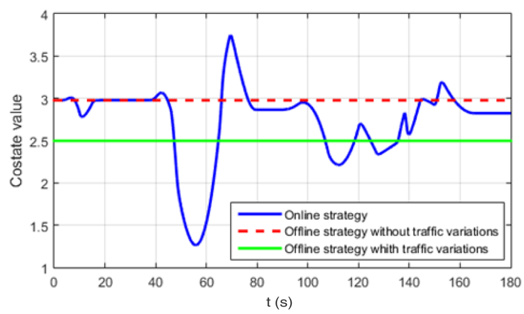
To deepen the analysis under more realistic and exhaustive test conditions, the online energy management strategy is evaluated over the FTP-75 (Federal Test



(a)

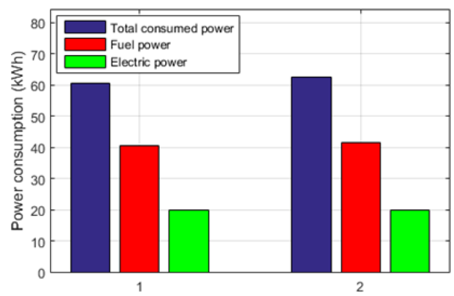


(b)

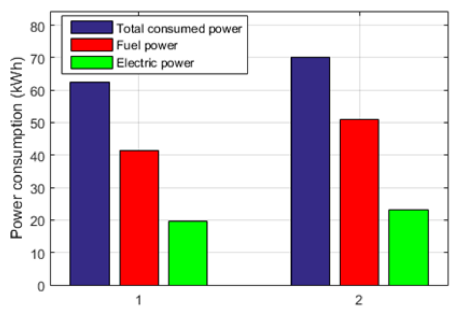


(c)

Figure 9. Simulation results of the online energy management strategy: (a) Torque distribution profile; (b) *SOE* profile; (c) Costate progress.



(a)



(b)

Figure 10. Power economy comparison: (a) Offline strategy (results set "1") versus online strategy (results set "2"); (b) Online strategy (results set "1") versus rules based strategy (results set "2").

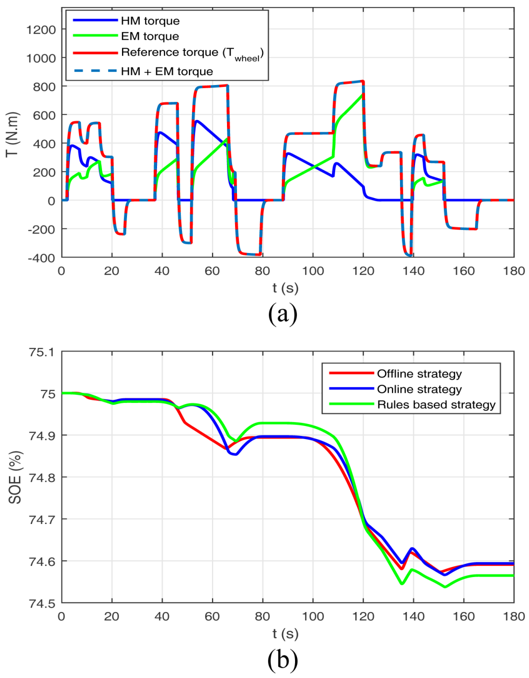


Figure 11. Simulation results of the rules based energy management strategy: (a) Torque distribution profile; (b) *SOE* trajectory.

Procedure) normalized driving cycle. This cycle includes acceleration and deceleration phases in urban and suburban environments. It thus constitutes an interesting study support for evaluating the performance of the energy management strategy in the different phases of operation of the real hybrid bus. The results obtained from this test are presented in Figure 12.

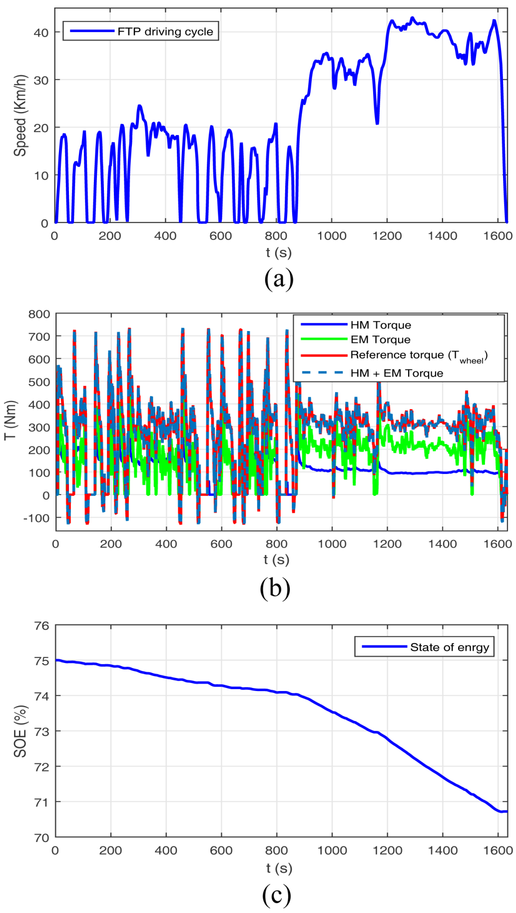


Figure 12. Simulation results of the online energy management strategy on the FTP-75 driving cycle: (a) Driving cycle; (b) Torque distribution profile; (c) *SOE* trajectory.

4.2. Validation Test under TruckMaker Software

The implementation of the energy management strategies discussed in this paper is carried out over the BUSINOVA TruckMaker simulator (cf. Figure 13) in order to investigate their performance in a test platform which reproduces accurately the real operating behavior of the bus. The video showing this overall demonstration using TruckMaker is available from this link: https://goo.gl/yc4rC0.

Figures 14 (a) and 15 (a) give an outline of the reference speed and the actual bus speed when the proposed offline and adaptive online energy management strategies are applied to calculate the optimal power split. It is to be noted that the bus speed reference is a simulation input signal defined by the user. Due to the effects of uncertainties on the future driving conditions, the shapes of the speed and torque curves are different in offline and online application cases (i.e., Figures 14 and 15 respectively). However, in both cases the control objectives must be achieved by ensuring a good tracking of the reference values. This implies the need for a good adaptability of the algorithm

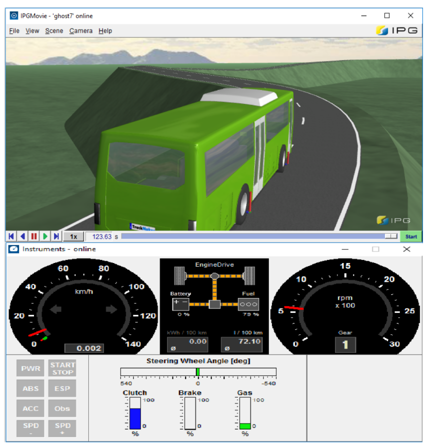
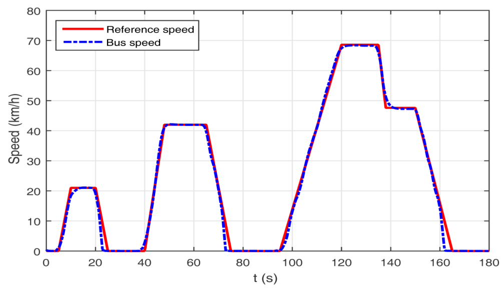
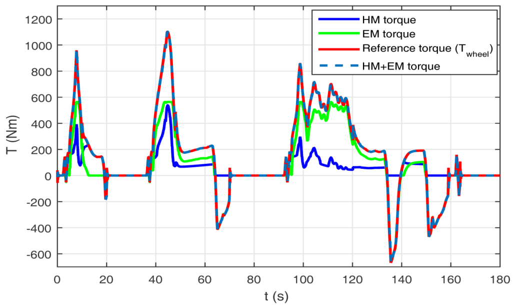


Figure 13. TruckMaker test platform.

especially in online use of the energy management strategy. As one can observe in Figures 14 (a) and 15 (a), the bus speed follows the reference speed curve with only a small

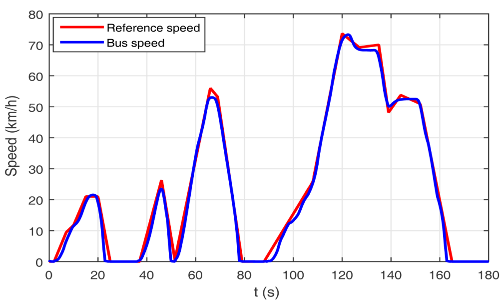


(a)

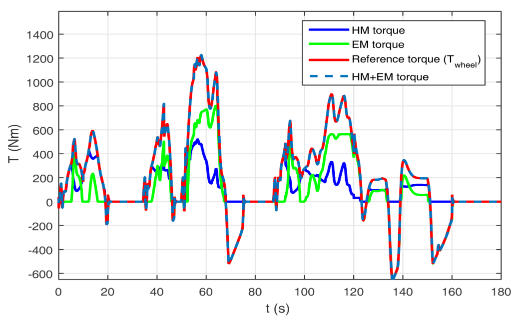


(b)

Figure 14. Speed and torque profiles from the offline strategy.



(a)



(b)

Figure 15. Speed and torque profiles from the adaptive online strategy.

tracking error. This observed speed tracking error is due mainly to the long response time of the motors which are, on top of that, not precisely modeled due to the motors nonlinearities. The sum of the motors reference torque curves calculated by both energy management strategies (cf. Figures 14 (b) and 15 (b)) closely follows the total torque demand (reference) at the wheels generated by the TruckMaker inverse dynamic model of the bus. The dynamic limits of the motors defined during the synthesis of the energy management strategy are also respected as can be seen in these figures.

5. CONCLUSION

An overall methodology of an energy management strategy is introduced in this paper by adapting the Pontryagin's minimum principle based resolution procedure of the energy minimization problem to plug-in multi hybrid electric bus. The proposed approach combines the system's equations with the analytical formulation of the control objectives to determine at each instant the optimal value of the control variable which minimizes the power consumption of the hybrid bus. Using the available knowledge of the future driving conditions, the optimization problem is solved, at first, offline in order to have the global optimal solution. This offline method is effective only if the pattern of the driving cycle is well known. However, in practice, the available driving information is usually uncertain because of the variation of driving conditions. In this case, the offline energy management strategy cannot satisfy the final condition on the SOE while using a constant costate, which represents the main limitation of this strategy. To address this issue, an online strategy has been developed to adapt the costate value based on the feedback from the current battery SOE value in order to take into account uncertainties on the knowledge of the driving cycle. The proposed adaptive strategy, which can be implemented easily in real-time, needs low calibration effort and it can offer accurate tuning of the costate even in the presence of large uncertainties on the future driving conditions. The implementation of this novel approach on the actual bus is planned to be done in near future. In future works, additional optimization criterion such as: Reduction of battery aging and pollutant emissions will be added to the already existing optimal control algorithm. The tradeoff between the different optimization criteria will be investigated in order to achieve the most efficient drivetrain operation.

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