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Safe cooperative merging strategy for platoon forming by a constrained multi-vehicle system

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Abstract— **We propose a kinematic motion strategy which allows performing a sequential merging of a set of curvature-constrained wheeled vehicles in order to obtain a platoon formation – one of the basic motion tasks defined for intelligent autonomous vehicles. The proposed design methodology is motivated by the lining-up maneuvering characterizing the N-trailer structures. The main emphasis has been paid on the safety guarantees of the dynamic navigation of the fleet of vehicles. In this context, we provide formal conditions under which all the vehicles during the maneuvers keep their motion in a prescribed road width, their longitudinal velocities and motion curvatures are constrained to the prescribed bounded ranges, and the minimal safe inter-vehicle distances are not violated. Formal considerations have been complemented by exemplary results of numerical simulations for a set of five vehicles.**

I. Introduction

The cooperative control design for a group of autonomous vehicles constitutes one of the fundamental issues in the multi-robot systems [1]. In the field of intelligent transportation, the merging for platoon-forming maneuver is among the basic and most challenging tasks defined for intelligent vehicles in public transportation (e.g., in urban areas or on highways) and in freight transportation [3], [4], [8]. The key factors which determine practical applicability of any multi-vehicle control system, apart from its effectiveness, are scalability (with respect to a number of vehicles), reliability, and safety guarantees of the maneuvers. Numerous works have been published so far in the robotic and control literature [17], [7], [10], [12] as well as in the intelligent transportation literature [14], [5] paying attention on the mentioned factors. In the context of control design for intelligent autonomous vehicles, much attention was paid on the platoon-control problem in the context of adaptive cruise control system addressing such issues like string stability, closed-loop robustness, control performance, and also safety of maneuvers [13], [18], [2], [15], [19], [16]. Platooning is a very attractive way of transportation which allows reducing energy consumption by the vehicles, reducing a traffic congestion, and providing safer and more comfortable driving with the help of better coordination between the vehicles. It is worth to mention here three high-impact initiatives devoted to this topic - the pioneering PATH project, the European SARTRE project, and recently launched the Grand Cooperative Driving Challenge which addresses, among other things, the cooperative merging of vehicle platoons. Despite the mentioned endeavors and numerous interesting results achieved so far, several important challenges still remain to be investigated due to high complexity and dimensionality of the problem [5], [10]. From the practical viewpoint, one still seeks relatively simple control approaches which would be scalable, distributed in nature, addressing constraints imposed on the vehicles, and simultaneously providing strict (analytical) conditions guaranteeing safe motion of the multi-vehicle system during various scenarios of maneuvering.

In this paper, a cooperative motion strategy is proposed which allows a set of arbitrary number of connected autonomous vehicles merging and platoon forming in a safe manner. Safety is preserved by keeping the upper and lower bounds of velocities and motion curvatures of the vehicles in a prescribed ranges, and by ensuring no collisions between the vehicles with their positions staying within a road of a given width. By using the smooth transitions between the control stages, it is also possible to limit the upper bounds of absolute accelerations of the vehicles during the maneuvers. Application of the control design methodology inspired by the lining-up maneuvering with the N-trailer vehicles (recently investigated in [9]), leads to a simple and scalable kinematic control strategy, with low computational complexity (also with respect to the number of maneuvering vehicles), and provides strict analytical conditions guaranteeing safety of nominal maneuvers. Implementation of the proposed control policy does not require planning of any reference trajectories for the vehicles (in contrast to the popular approach – see, e.g., [4], [11]), is not based on any numerical optimization (opposed, e.g., to [10]), and needs measuring of only the actual positions and orientations of the vehicles. All the mentioned properties seem to be especially beneficial in the context of embedded control solutions demanding low computational power from the vehicles' onboard computers. Therefore, the proposed solution seems to be competitive when related to other strategies known from the literature.

The rest of the paper is organized as follow. Section II is devoted to modeling of the studied multi-vehicle system. Section III contains the main assumptions and a formulation of the control problem under consideration. The proposed control strategy is described and formally justified in Section IV. Selected simulation results, illustrating the nominal control performance, together with comments are presented in Section V.

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II. Modeling the multi-vehicle system

Let us consider a set $\mathcal{S}$ of M autonomous vehicles moving along a straight road of a prescribed width $D > 0$ (see Fig. 1). The vehicles are labeled by V_j with only even subscripts (including zero), i.e., $j \in \mathbb{Z}_e \triangleq \{0, 2, 4, \ldots, 2i-2, 2i, 2i+2, \ldots, 2M-2\}$. Kinematics of the V_{2i} vehicle body will be modeled (using $\mathrm{s}\gamma \equiv \sin\gamma$, $\mathrm{c}\gamma \equiv \cos\gamma$) as

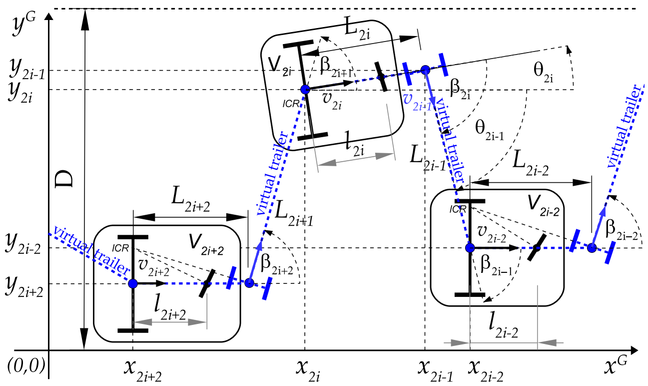

Fig. 1. Configuration variables and kinematic parameters of a multi-vehicle system virtually connected (through virtual trailers highlighted in blue) in the N-trailer structure while moving in the fixed (global) frame $\{G\}$; the angles θ_{2i} and θ_{2i-1} are defined with respect to a positive semi-axis x^G

$$\dot{\boldsymbol{q}}_{2i} = \boldsymbol{G}(\theta_{2i})\boldsymbol{u}_{2i} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \mathrm{c}\theta_{2i} & \mathrm{s}\theta_{2i} \end{bmatrix}^\top \boldsymbol{u}_{2i}, \qquad (1)$$

where $\boldsymbol{q}_{2i} = [\theta_{2i}\ x_{2i}\ y_{2i}]^\top \in (-\pi, \pi] \times \mathbb{R}^2$ is a vehicle-body configuration with orientation angle θ_{2i} and positional coordinates (x_{2i}, y_{2i}), while

$$\boldsymbol{u}_{2i} = \begin{bmatrix} \omega_{2i} \\ v_{2i} \end{bmatrix} = \begin{bmatrix} (\tan\delta_{2i}/l_{2i})v_{2i} \\ v_{2i} \end{bmatrix} \in \mathcal{U} \subset \mathbb{R}^2 \qquad (2)$$

is a kinematic control input comprising the angular velocity ω_{2i} of a vehicle body and the longitudinal velocity v_{2i} of a midpoint (x_{2i}, y_{2i}) of the rear fixed (non-steerable) wheels. The term δ_{2i} is a steering angle of a front wheel which is assumed to be accurately controlled by the vehicle's on-board servo guaranteeing that $\forall\, t \geq 0\ \ \delta_{2i}(t) = \arctan(l_{2i}\omega_{2i}(t)/v_{2i}(t))$. The parameter $l_{2i} > 0$ denotes a distance between the rear wheels' axle and a steering wheel of the vehicle V_{2i} (cf. Fig. 1).

The order of vehicles in set $\mathcal{S}$ is determined by the subscripts $j \in \mathbb{Z}_e$ assigned according to vehicles' coordinates along the x^G axis, that is, a vehicle V_{2i+2} is a subordinate of a vehicle V_{2i} if $x_{2i+2} < x_{2i}$. A leading vehicle, denoted as V_0, is selected as a most forefront vehicle from the set $\mathcal{S}$. If for any two vehicles V_{j_1} and V_{j_2} holds $x_{j_1} = x_{j_2}$, then their ordering can result, e.g., from their initial distances to the leader, that is, $j_1 > j_2$ if $d_{j_1,0}(0) > d_{j_2,0}(0)$ where

$$d_{i_1,i_2}(t) \triangleq \sqrt{(x_{i_1}(t) - x_{i_2}(t))^2 + (y_{i_1}(t) - y_{i_2}(t))^2} \qquad (3)$$

is a distance between any two vehicles V_{i_1} and V_{i_2}, or from other particular preference.

In Section IV, we are going to adopt a methodology originally applied to the N-trailer vehicles [9]. Therefore, for the purposes of motion strategy design, let us treat the multi-vehicle system as a virtual N-trailer, where the leading vehicle V_0 is treated as a tractor and all other vehicles (the followers) are treated as trailers of some prescribed lengths $L_{2i} > 0$. Introduce also the *virtual trailers* (denoted in blue in Fig. 1) of lengths

$$L_{2i-1}(t) \triangleq \sqrt{A_i^2(t) + B_i^2(t)}, \qquad (4)$$

and of the orientation angles

$$\theta_{2i-1}(t) \triangleq \mathrm{Atan2}\left(B_i(t), A_i(t)\right) \in (-\pi, \pi], \qquad (5)$$

where $A_i(t) = x_{2i-2}(t) - x_{2i}(t) - L_{2i}\mathrm{c}\theta_{2i}(t)$ and $B_i(t) = y_{2i-2}(t) - y_{2i}(t) - L_{2i}\mathrm{s}\theta_{2i}(t)$. Note that the lengths (4) (and angles (5)) are computed as functions of, generally, time-varying arguments. Thus, in contrast to parameters L_{2i}, the lengths (4) are not prescribed but must be computed on-line. Following [9], one shall expect that longer lengths L_{2i} and L_{2i-1} will imply longer duration of the merging maneuver, which is performed with smaller absolute angular velocities of the vehicles. Furthermore, longer lengths will make the steady inter-vehicle distances larger in a formed platoon.

The virtual trailers V_l (their lengths and orientation angles as well) are all labeled with odd subscripts $l \in \mathbb{Z}_o \triangleq \{1, 3, 5, \ldots, 2i-1, 2i+1, \ldots, 2M-3\}$. They virtually join the real vehicles forming together a virtual kinematic chain of the Standard N-Trailer (SNT) [9]. The virtual joint-angles β_j and β_l, with $j \in \mathbb{Z}_e$ and $l \in \mathbb{Z}_o$, are defined as follows (see Fig. 1)

$$\forall\, i \geq 1 \quad \beta_{2i} \triangleq \theta_{2i-1} - \theta_{2i}, \quad \beta_{2i-1} \triangleq \theta_{2i-2} - \theta_{2i-1}, \quad (6)$$

and they can be collected in a vector

$$\boldsymbol{\beta}(t) = [\beta_1(t)\ \beta_2(t)\ \ldots\ \beta_N(t)]^\top \in \mathbb{T}^N, \qquad (7)$$

where $N = 2M - 2$ for a set of M vehicles. For the virtual SNT kinematic structure from Fig. 1 one can easily derive for any $i \in \mathbb{Z}_o \cup \mathbb{Z}_e \setminus \{0\}$ the following velocity transformation

$$\begin{bmatrix} \omega_i \\ v_i \end{bmatrix} = \begin{bmatrix} 0 & \mathrm{s}\beta_i/L_i \\ 0 & \mathrm{c}\beta_i \end{bmatrix} \begin{bmatrix} \omega_{i-1} \\ v_{i-1} \end{bmatrix} \ \Leftarrow\ \boldsymbol{u}_i = \boldsymbol{J}_i(\beta_i)\boldsymbol{u}_{i-1} \quad (8)$$

which is valid when $L_i = \mathrm{const}$ (see [9]), and where any v_l, with an odd index $l \in \mathbb{Z}_o$, denotes a longitudinal velocity of the point (x_l, y_l), which is co-linear to the length L_l of the virtual trailer (see Fig. 1 for $l = 2i - 1$).

III. Prerequisites and problem statement

A. Basic assumptions

A1: Initially, i.e., for $t \in [0, t_0^*)$ all the vehicles move with some finite constant velocity $v_m > 0$ within a road of width $D > 0$, parallel and sufficiently far from the road borders, that is, $\forall\, i \quad y_{2i}(t) \in [\varepsilon_{2i}, D - \varepsilon_{2i}]$, $\varepsilon_{2i} > 0$, $\theta_{2i}(t) = 0$, $v_{2i}(t) = v_m$, and $d_{2i,2i-2}(t) \geq d_{\min}$ for some prescribed minimal distance $d_{\min} > 0$.

A2: The leading vehicle V_0 persistently moves along a straight lane, that is, $\forall\, t \geq 0\ \ y_0(t) = y_0(0) = y_0$ and $\theta_0(t) \equiv \theta_0(0) = 0$.

A3: The steering angle δ_{2i} of any vehicle V_{2i} is constrained to some prescribed range $[-\Delta_{2i}, \Delta_{2i}]$ with $\Delta_{2i} < \pi/2$.

A4: The configuration $\boldsymbol{q}_{2i}(t)$ of any vehicle V_{2i} is measurable on-line, and any vehicle V_{2i} can bi-directionally communicate at least with its neighbors, i.e., with vehicles V_{2i-2} and V_{2i+2}, to exchange the necessary data (see Remark 3 in Section IV-B).

Assumption A1 describes a typical situation where a set of coordinated vehicles initially moves in steady conditions (within some finite time interval $[0, t_0^*)$) along a road (e.g., along various lanes on a highway) in some physical separation of their bodies (greater than $d_{\min}$), which is not necessarily *safe* for the merging-for-platoon-forming maneuvers but only ensures initial collision-free motion. A2 indicates that the leading vehicle determines a constant reference y-location of the expected platoon formation within the road width. Assumption A3 is natural (due to mechanical construction) in any real car-like vehicle. Moreover, A3 corresponds to a limit imposed on a maximal possible absolute curvature $|\kappa_{2i}(t)|$ of the V_{2i} vehicle's motion. Finally, assumption A4 allows closing a feedback from the current vehicles' configurations, and exchange other data necessary in the cooperative control process between any vehicles in $\mathcal{S}$.

B. Control problem formulation

Let us introduce the y-positional errors

$$e_{2i}(t) \triangleq y_{2i-2}(t) - y_{2i}(t), \tag{9}$$

where $y_{2i-2}(t) = y_0$ for $i = 1$ denotes (upon A2) a constant reference y-coordinate determined by the leading vehicle V_0.

Problem 1 (Merging-for-Platoon-Forming (MPF) task): For a given set $\mathcal{S}$ of M vehicles satisfying assumptions A1-A4, a control problem is to find the bounded control laws $\boldsymbol{u}_{2ic}(t) = [\omega_{2ic}(t)\ v_{2ic}(t)]^\top$ which, when applied into kinematics given in (1) by taking $\boldsymbol{u}_{2i}(t) := \boldsymbol{u}_{2ic}(t)$ for all $i \in \{0, 1, 2, \ldots, M-1\}$, merges all the vehicles in a terminal platoon in the sense that $\forall\, i$

$$e_{2i}(t) \overset{t\to\infty}{\longrightarrow} 0 \ \wedge\ \theta_{2i}(t) \overset{t\to\infty}{\longrightarrow} 0 \ \wedge\ (\boldsymbol{u}_{2i}(t) - \boldsymbol{u}_{0c}(t)) \overset{t\to\infty}{\longrightarrow} \mathbf{0}, \tag{10}$$

preserving the prescribed vehicles order by keeping $x_{2i-2}(t) - x_{2i}(t) > 0$ for $t \to \infty$, and satisfying $\forall\, i$ the following *safety guarantees* and constraints:

(g1) $\forall\, t > 0\ v_{2ic}(t) \in [v_m, v_M)$,
(g2) $\forall\, t > 0\ |\kappa_{2ic}(t)| \triangleq |\omega_{2ic}(t)/v_{2ic}(t)| \le (\tan \Delta_{2i})/l_{2i}$,
(g3) $\forall\, t > 0\ y_{2i}(t) \in (0, D)$,
(g4) $\forall\, t > 0\ d_{2i,2i-2}(t) \ge d_{\min}$,

where $v_M > v_m$ is a prescribed (finite) upper bound of the admissible longitudinal velocity, while $d_{\min} > 0$ is a prescribed minimal admissible distance between the vehicles.

By condition (10), we expect the vehicles to terminally converge (in the order prescribed at the beginning of a control process) to a common straight line determined by the leading vehicle V_0, moving terminally with the same longitudinal velocity v_{0c}. The latter requirement ensures constant terminal (steady) inter-vehicle distances in a platoon. We do not require the steady inter-vehicle distances to be equal, since this objective can be efficiently achieved by subsequent usage of the platoon-control methods widely discussed in the literature – see, e.g., [8]. On the other hand, we require satisfaction of four safety guarantees and constraints: limiting longitudinal velocities to the prescribed admissible range (g1), limiting the maximal admissible motion curvature to a prescribed range (g2) corresponding to assumption A3, keeping the vehicles positions within the road width (g3), and collision-free motion (g4) of any two neighboring vehicles in the set $\mathcal{S}$.

Remark 1: The MPF task has been formulated in the space of velocities, since the vehicles are modeled solely on a kinematic level. In this context, the control problem under consideration shall be read in terms of a desired *nominal motion strategy*.

IV. Control strategy description

A. Control stages and the MPF control law

In order to meet all the safety guarantees and constraints formulated in Problem 1 we propose to divide a control process into two main subsequent stages:

- stage (S1) responsible for a preparation of the set $\mathcal{S}$ by ordering the vehicles (assigning them numbers) and stretching the vehicle set along a road to ensure sufficiently large (safe) distances between the vehicles and to achieve favorable (in view of the next control stage) configuration of the virtual N-trailer chain,
- stage (S2) responsible for sequential merging of the consecutive vehicles into a platoon.

From now on, we assume that a transition from (S1) to (S2) is allowed to be performed only once for a given vehicle in the whole control time horizon (the mono-stable transition), avoiding this way any potential chattering effect.

We propose a nominal motion strategy for the leading vehicle V_0 by defining

$$\boldsymbol{u}_{0c}(t) \triangleq \begin{bmatrix} 0 \\ v_m + (v_M - v_m)\tanh\left(\alpha(\tau_\alpha)\,\|\boldsymbol{\beta}(t)\|\right) \end{bmatrix}, \tag{11}$$

where $\alpha : \mathbb{R} \to [0, 1]$ is a smooth *activation function*

$$\alpha(\tau_\alpha) \triangleq \frac{\lambda(\tau_\alpha)}{\lambda(\tau_\alpha) + \lambda(1 - \tau_\alpha)} \tag{12}$$

with

$$\lambda(z) \triangleq \begin{cases} 0 & \text{for } z \le 0, \\ \exp(-1/z) & \text{for } z > 0, \end{cases}$$

such that $\alpha(\tau_\alpha \le 0) = 0$ and $\alpha(\tau_\alpha \ge 1) = 1$, while $\tau_\alpha \triangleq (t - t_0^*)/T_\alpha$ is the scaled time-variable with a prescribed scaling factor $0 < T_\alpha < \infty$. The prescribed (finite) delay time t_0^* determines beginning of the control stage (S1). It is worth to note that velocity $v_{0c}(t)$ defined by (11) is a convex combination of boundary velocities v_m and v_M, such that $v_{0c} = v_m$ if $\alpha(\tau_\alpha)\,\|\boldsymbol{\beta}(t)\| = 0$ and $v_{0c} \to v_M$ if $\|\boldsymbol{\beta}\| \to \infty$.

A nominal motion strategy for any vehicle V_{2i}, $i \in \{1, 2, \ldots, M-1\}$, is defined in an iterative form

$$\boldsymbol{u}_{2ic}(t) = \begin{bmatrix} \omega_{2ic}(t) \\ v_{2ic}(t) \end{bmatrix} \triangleq \begin{bmatrix} s_{2i}(\tau_s, w_{2i}) \cdot \tilde{\omega}_{2ic}(t) \\ \max\{v_m, \tilde{v}_{2ic}(t)\} \end{bmatrix}, \tag{13}$$

where

$$\begin{bmatrix}\tilde{\omega}_{2ic}(t)\\ \tilde{v}_{2ic}(t)\end{bmatrix} \triangleq \boldsymbol{J}_{2i}(\beta_{2i}(t))\boldsymbol{J}_{2i-1}(\beta_{2i-1}(t))\boldsymbol{u}_{2i-2c}(t) \quad (14)$$

with matrices $\boldsymbol{J}_{2i}(\beta_{2i})$ and $\boldsymbol{J}_{2i-1}(\beta_{2i-1})$ of the forms resulting from transformation (8). Note that matrix $\boldsymbol{J}_{2i}$ is computed using the prescribed parameter L_{2i}, while matrix $\boldsymbol{J}_{2i-1}$ must be computed with the current length (4) updated on-line. The term $s_{2i}: \mathbb{R}\times\{0,1\}\to[0,1]$ used in (13) is a *transition operator*

$$s_{2i}(\tau_s, w_{2i}) \triangleq \begin{cases} 0 & \text{if} \quad w_{2i}=0,\\ \alpha(\tau_s) & \text{if} \quad w_{2i}=1,\end{cases} \quad (15)$$

where $\alpha(\tau_s)$ is the activation function determined by (12). The argument w_{2i} allows forcing (at some time instant t^*_{2i}) the function s_{2i} to transit from zero to unity, simultaneously determining a transition from the control stage (S1) to the control stage (S2) for the vehicle V_{2i}. The first argument $\tau_s \triangleq (t-t^*_{2i})/T_s$ is a scaled time-variable with a prescribed scaling factor $0 \le T_s < \infty$; $w_{2i}\in\{0,1\}$ is a bi-valued decision variable which governs the transition process, that is, it either keeps the function s_{2i} at zero if $w_{2i}=0$ or releases the transition of s_{2i} from zero to unity in a finite time interval T_s if $w_{2i}=1$. The decision variable w_{2i} will be very useful to release a transition from control stage (S1) to stage (S2) for vehicle V_{2i} only after satisfaction of all the necessary conditions guaranteeing safe maneuvers in stage (S2), see Section IV-B. The time instant t^*_{2i} used in definition of τ_s represents a moment when decision variable w_{2i} changes its value from zero to one. For the set of M vehicles we have the number of $M-1$ decision variables $\{w_2, w_4, w_6, \ldots, w_{2M-2}\}$ (excluding the leading vehicle), which are manipulated by the control system.

Remark 2: Activation function $\alpha(\cdot)$ used in (11) allows the leading vehicle accelerate smoothly in the case of a non-zero norm $\|\boldsymbol{\beta}(t)\|$ already at the beginning of a control process, that is, at $t=t^*_0$. Transition operator s_{2i} used in (13) leads to a unidirectional transition from the zero angular velocity, necessary for control stage (S1), to the value $\tilde{\omega}_{2ic}(t)$ required in stage (S2) during the MPF maneuvers. Note that by increasing the scaling factor T_s in the definition of τ_s used in (15) one can make the transitions process smooth, which leads to smaller angular accelerations. On the other hand, by selecting the factor T_s smaller, one makes the transition sharper, up to the instantaneous *switching transition* at the limit for $T_s=0$.

B. Motion strategy formulation with safety conditions

Proposition 1: The MPF control law, defined by (11) and (13), applied to the control inputs of kinematics (1), by forcing, respectively, $\boldsymbol{u}_0(t) := \boldsymbol{u}_{0c}(t)$ and $\boldsymbol{u}_{2i}(t) := \boldsymbol{u}_{2ic}(t)$ for every $i\in\{1,2,\ldots,M-1\}$, solves the Problem 1 if the pre-defined parameters v_m, v_M, and selected L_{2i} satisfy

$$\frac{v_M}{v_m L_{2i}} \le \frac{\tan\Delta_{2i}}{l_{2i}}, \quad (16)$$

and if for every vehicle V_{2i}, $i\in\{1,2,\ldots,M-1\}$, one applies the *switching transition* ($T_s := 0$) with the decision variable $w_{2i}=0$ for $t\in[0,t^*_{2i})$, and $w_{2i}=1$ for $t\ge t^*_{2i}$ starting at $t=t^*_{2i}$ when simultaneously:

(c1) $|\beta_k(t^*_{2i})| < \frac{\pi}{2}$ for $k\in\{1,2,\ldots,2i,2i+1,2i+2\}$,
(c2) $v_{2i-2c}(t^*_{2i}) > v_{2ic}(t^*_{2i}) > v_m$ (acute inequalities),
(c3) $L_{2i-1}(t^*_{2i}) \ge L_{2i}/[\zeta \mathrm{c}\beta_{2i-1}(t^*_{2i})]$, for some $\zeta\in(0,1)$, and $2L_{2i}L_{2i-1}(t^*_{2i})\mathrm{c}\beta_{2i}(t^*_{2i}) + L^2_{2i-1}(t^*_{2i}) \ge d^2_{\min} - L^2_{2i}$,
(c4) $L_{2i+1}(t^*_{2i}) \ge \frac{d_{\min}+L_{2i-1}(t^*_{2i})(1-\mathrm{c}\beta_{2i}(t^*_{2i}))-L_{2i+2}}{\mathrm{c}\beta_{2i+2}(t^*_{2i})}$,
(c5) $\sum_{k=1}^{i-1}|e_{2k}(t^*_{2i})| \le \epsilon_{2i}$, taking $\epsilon_{2i}>0$ as a prescribed sufficiently (*negligibly*) small constant such that

$$2\epsilon_{2i} + (L_{2i}/v_m)\bar{\sigma}_{2i} < \varepsilon_{2i}, \quad (17)$$

with ε_{2i} introduced in A1, and $\bar{\sigma}_{2i} = \sup_{t\ge t^*_{2i}} \sigma_{2i}(t)$ being a supremum of a non-negative function

$$\sigma_{2i}(t) = \left[\frac{L_{2i-1}(t)}{L_{2i}}\,|\mathrm{c}\beta_{2i}(t)\mathrm{c}\beta_{2i-1}(t)\mathrm{s}\theta_{2i-1}(t)| + |\mathrm{s}\theta_{2i-2}(t)|\right] v_{2i-2c}(t). \quad (18)$$

Under conditions (c1)-(c5), the steady inter-vehicle distances in a platoon will satisfy: $d_{2i,2i-2}(\infty) \ge L_{2i-1}(t^*_{2i}) + L_{2i}$.

Simultaneous satisfaction of conditions (c1) to (c5) determines a time-instant t^*_{2i} of transition from control stage (S1) to control stage (S2) for the vehicle V_{2i}, which should guarantee safe MPF maneuver performed with this vehicle V_{2i}. Condition (c2) requires that a longitudinal velocity of a preceding vehicle V_{2i-2} is strictly greater than a longitudinal velocity of vehicle V_{2i}, while the latter is strictly greater than the lower bound v_m. Conditions (c3) and (c4) determine how much the distances between neighboring vehicles $V_{2i-2}, V_{2i}, V_{2i+2}$ should be enlarged during the control stage (S1) to guarantee safe maneuvering with the vehicle V_{2i} in stage (S2) in the sense of not colliding with the preceding vehicle (V_{2i-2}) and the following vehicle (V_{2i+2}), respectively. The first condition in (c5) corresponds to (*almost*) finishing of the MPF maneuvers by all the preceding vehicles from V_2 to V_{2i-2} (of course, for V_0 we can say that the leading vehicle trivially and exactly 'finishes' MPF maneuver already at t^*_0, i.e., the sum in (c5) for $i=1$ should be taken as equal to zero). (c5) indicates also that the proposed MPF strategy will be performed sequentially by particular vehicles $V_2, V_4, \ldots, V_{2M-2}$, started at time instants $t^*_2 \le t^*_4 \le \ldots \le t^*_{2M-2}$, respectively. Finally, the conservative condition (17) allows satisfying the guarantee (g3) – it determines how far from the road boundaries (by the value of ε_{2i} introduced in assumption A1) the vehicle V_{2i} should initially stay to not violate (g3) during the MPF. Note that supremum $\bar{\sigma}_{2i}$ in (17) can be replaced by the (more conservative) upper bound $\bar{\bar{\sigma}}_{2i} = v_M[(\sup_{t\in T_2} L_{2i-1}(t)/L_{2i})+1]$, where for the case of *switching transition* from stage (S1) to (S2) holds $\sup_{t\in T_2} L_{2i-1}(t) = L_{2i-1}(t^*_{2i})$, see Section IV-C.

Remark 3: According to the proposed control policy (11)-(14), one can infer which data are needed to be exchanged between the cooperating vehicles. Namely, at any time instant $t\ge t^*_0$ the leading vehicle has to receive current values of all the virtual joint angles $\beta_1(t),\ldots,\beta_N(t)$. On the other hand, every vehicle V_{2i} (for $i>0$) has to receive configurations $\boldsymbol{q}_{2i-2}(t)$ and $\boldsymbol{q}_{2i+2}(t)$, the computed velocity $\boldsymbol{u}_{c2i-2}(t)$, the

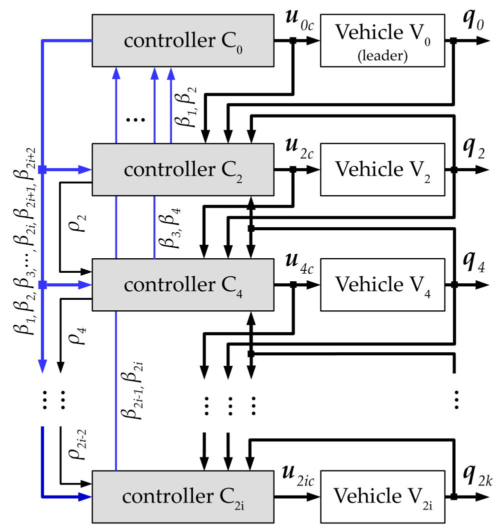

Fig. 2. A block scheme of the proposed MPF control system

value of $\rho_{2i-2}(t) := \sum_{k=1}^{i-1} |e_{2k}(t)|$, and the current values of virtual joint angles $\beta_1(t), \ldots, \beta_{2i+2}(t)$. A block scheme explaining a structure of the proposed control system has been presented in Fig. 2.

Remark 4: Proposition 1 has been formulated for the case of the (non-smooth) *switching transition* between stages (S1) and (S2). It is worth noting that Problem 1 can be effectively solved under conditions (c1)-(c5) using also the smooth transition by taking (even relatively large) $T_s > 0$. It will be illustrated by simulation results in Section V. By using the smooth transition, the maximal absolute values of angular accelerations can be made acceptably small leading to more smooth motion of the following vehicles. A formal analysis of the system behavior in the case of the smooth transition, however, requires further investigations.

C. Formal justification of Proposition 1

Let us provide formal arguments which justify the claim formulated in Proposition 1 by a (sketch) analysis of the closed-loop system for subsequent stages (S1) and (S2).

a) Arguments common for stages (S1) and (S2): We show first that guarantees (g1)-(g2) are satisfied in both control stages (S1) and (S2). Upon (11) it is clear by construction that $\omega_{0c}(t) \equiv 0$ and $v_{0c}(t) \in [v_m, v_M)$ because $v_{0c}(t) = v_M$ only if $\|\boldsymbol{\beta}(t)\| = \infty$ which is impossible in $\mathbb{T}^N$. Upon (13) and (14) it is clear by construction that $\inf_t |\omega_{2ic}(t)| = 0$, $\inf_t v_{2ic}(t) = v_m$. Furthermore, we can assess: $\sup_t |\omega_{2ic}(t)| \leq \sup_t |v_{2i-2c}(t)| / L_{2i}$ and also $\sup_t v_{2ic}(t) \leq \max\{v_m, \sup_t v_{2i-2c}(t)\}$. Thus, for $i = 1$ we have $\sup_t |\omega_{2c}(t)| \leq \sup_t |v_{0c}(t)| / L_2 < v_M/L_2$, and $\sup_t v_{2c}(t) \leq \max\{v_m, \sup_t v_{0c}(t)\} < \max\{v_m, v_M\} = v_M$; for $i = 2$ we have $\sup_t |\omega_{4c}(t)| \leq \sup_t |v_{2c}(t)| / L_4 < v_M/L_4$, and $\sup_t v_{4c}(t) \leq \max\{v_m, \sup_t v_{2c}(t)\} < \max\{v_m, v_M\} = v_M$. Proceeding for $i > 2$, one concludes:

$$\forall t \geq 0 \quad \omega_{0c}(t) = 0 \ \wedge \ |\omega_{2ic}(t)| \in [0, v_M/L_{2i}), \tag{19}$$

$$\forall t \geq 0 \quad v_{0c}(t) \in [v_m, v_M) \ \wedge \ v_{2ic}(t) \in [v_m, v_M) \tag{20}$$

for $i \in \{1, 2, \ldots, M-1\}$, which satisfy (g1). Next, the motion curvature $\kappa_{2ic}(t) = \omega_{2ic}(t)/v_{2ic}(t)$ is zero for vehicle V_0, while for other vehicles we can assess upon (19)-(20): $\sup_t |\kappa_{2ic}(t)| = (\sup_t |\omega_{2ic}(t)|)/(\inf_t v_{2ic}(t)) < v_M/(v_m L_{2i})$. Under condition (16), (g2) is satisfied.

b) Arguments for stage (S1): The duration of stage (S1) for the vehicle V_{2i} is $T_1 = [t_0^*, t_{2i}^*)$. In this stage $w_{2i} = 0$, thus $\omega_{2i}(t) := \omega_{2ic}(t) = 0$ for $t \in T_1$. By recalling (1) and assumption A1, it is clear that $\dot{y}_{2i}(t) \equiv 0$ for $t \in T_1$, and upon assumptions A1-A2 the guarantee (g3) it trivially satisfied. Next, let us show that conditions (c1)-(c4) can be satisfied in finite time in (S1). Upon the previous intermediate conclusions, and due to A1-A2, one can derive the following equations valid $\forall\, i \in \{1, \ldots, M-1\}$ within stage (S1):

$$\dot{\beta}_{2i}(t) = -\frac{\tilde{v}_{2i-2,2i}(t)}{L_{2i-1}(t)} \mathrm{s}\beta_{2i}(t), \quad \beta_{2i-1}(t) = -\beta_{2i}(t), \tag{21}$$

$$\dot{L}_{2i-1}(t) = \tilde{v}_{2i-2,2i}(t) \mathrm{c}\beta_{2i}(t), \tag{22}$$

where $\tilde{v}_{2i-2,2i} \triangleq v_{2i-2c} - v_{2ic}$ is a difference between the nominal velocities of vehicles V_{2i-2} and V_{2i}. Since $v_{0c}(t) > v_m$ and $v_{2c}(t) = \max\{v_m, v_{0c}(t)\mathrm{c}\beta_1(t)\mathrm{c}\beta_2(t)\} < v_{0c}(t)$ (acute inequality) for all $t > t_0^*$ if only $\mathrm{c}\beta_1(t)\mathrm{c}\beta_2(t) < 1$ (i.e., if vehicles V_0 and V_2 do not form a platoon yet), one concludes that $\tilde{v}_{0,2} > 0$ and upon (21) that $\beta_1(t)$ and $\beta_2(t)$ will have a tendency to converge toward zero in (S1). By the same argument, $L_1(t)$ (upon (22)) will start increasing if only $|\beta_2(t)|$ becomes less than $\pi/2$. Similar arguments can be formulated to show that $v_{2ic}(t) \leq v_{2i-2c}(t)$ for all $t \in T_1$. However, to obtain the condition $v_m < v_{2ic}(t) < v_{2i-2c}(t)$ for $i \in \{1, \ldots, M-1\}$, with both acute inequalities corresponding to (c2), one has to satisfy the following inequality $v_{2i-2c}\mathrm{c}\beta_{2i}\mathrm{c}\beta_{2i-1} = v_{0c} \prod_{k=1}^{2i} \mathrm{c}\beta_k > v_m$ which, according to (11), leads to the condition

$$\tanh(\alpha(\tau_\alpha)\,\|\boldsymbol{\beta}\|) + \frac{1 - \prod_{k=1}^{2i} \mathrm{c}\beta_k}{\prod_{k=1}^{2i} \mathrm{c}\beta_k} < \frac{v_M}{v_m}. \tag{23}$$

Since $0 \leq \tanh(\alpha(\tau_\alpha)\,\|\boldsymbol{\beta}(t)\|) < 1$ for all $t \geq t_0^*$ and $(v_M/v_m) > 1$ (by definition), it is always possible to meet (23) in a finite time thanks to the convergence tendency for angles $\beta_k(t)$ resulting from equations (21). Thus, satisfaction of required conditions (c1) and (c2) is possible within a finite time interval in stage (S1). Furthermore, meeting (c1) and (c2) implies positive right-hand side of (22). As a consequence, all the lengths $L_{2i-1}(t)$ with odd indexes will increase by an integral action. Therefore, by keeping the system in stage (S1) through the sufficiently long finite time interval T_1 it is possible to increase the lengths $L_{2i-1}(t)$ and $L_{2i+1}(t)$ to the values imposed by conditions (c3)-(c4). A rate of change for the lengths $L_{2i-1}(t)$ proportionally depends on the velocity difference $\tilde{v}_{2i-2,2i}(t)$, which can be increased by admitting a wider range $[v_m, v_M)$ of the admissible longitudinal velocities introduced in (g1).

Finally, let us show that guarantee (g4) is also satisfied within stage (S1). One can easily check that in stage (S1)

$$\dot{d}_{2i,2i-2}(t) = \frac{x_{2i-2}(t) - x_{2i}(t)}{d_{2i,2i-2}(t)} \tilde{v}_{2i-2,2i}(t), \tag{24}$$

where $\tilde{v}_{2i-2,2i} \triangleq v_{2i-2c} - v_{2ic}$, and $d_{2i,2i-2}$ is the Euclidean distance between vehicles V_{2i} and V_{2i-2}. Since $x_{2i-2}(0) - x_{2i}(0) \geq 0$ (by the postulated rule of vehicles numbering) and $\tilde{v}_{2i-2,2i}(t) \geq 0$ for all $t \geq 0$ (by the arguments formulated so far), it is clear from (24) that the distance $d_{2i,2i-2}(t)$ cannot decrease during stage (S1). In view of assumption A1, one guarantees (g4). Moreover, when $\tilde{v}_{2i-2,2i}(t)$ becomes strictly positive (it happens in finite time by satisfaction of (23)), the distance $d_{2i,2i-2}(t)$ will become increasing.

c) Arguments for stage (S2): In the case of the *switching transition* from (S1) to (S2), a duration of the control stage (S2) for the vehicle V_{2i} will be represented by the range $T_2 = [t^*_{2i}, \infty)$. Switching to the control stage (S2) at $t = t^*_{2i}$ for the vehicle V_{2i} means that conditions (c1)-(c4) are satisfied, and additionally (c5) holds. The latter means that either $V_{2i-2} = V_0$ is the leading vehicle if $i = 1$ (in this case the sum in (c5) is exactly zero, and $\theta_0(t) \equiv 0$ by A2), or (if $i \geq 2$) all the preceding vehicles $V_0, \ldots, V_{2i-2}$ have just (*almost*) finished their MPF maneuvers (cf. (10)). In particular, satisfaction of (c5) together with assumption A2 (and according to the results provided in [9]) allows one to conclude

$$\forall\, t \geq t^*_{2i} \quad \theta_{2i-2}(t) \approx 0 \quad \wedge \quad \theta_{2i-2}(t \to \infty) \to 0. \quad (25)$$

Hereafter, we also assume that condition (c2) when satisfied at $t = t^*_{2i}$ is also preserved for any finite $t \in T_2$ (see Remark 5). Under this assumption, one can take $v_{2ic}(t) = \tilde{v}_{2ic}(t) = v_{2i-2c}(t)\mathrm{c}\beta_{2i}(t)\mathrm{c}\beta_{2i-1}(t)$ (see (13)-(14)), which allows one to show that

$$\dot{L}_{2i-1}(t) = [1 - s_{2i}(t)]v_{2i-2c}(t)\mathrm{s}^2\beta_{2i}(t)\mathrm{c}\beta_{2i-1}(t). \quad (26)$$

Thus, for all $t \in T_2$ (where $s_{2i}(t) \equiv 1$) holds

$$\dot{L}_{2i-1}(t) \equiv 0 \;\Rightarrow\; L_{2i-1}(t) = L_{2i-1}(t^*_{2i}) = \text{const}. \quad (27)$$

In view of (27) and (25), and since $v_{jc}(t) > 0$ for all $t \geq 0$ and all $j \in \mathbb{Z}_e$, one can treat now the subset of two vehicles, V_{2i} and V_{2i-2}, as a virtual tractor-trailer system (more precisely: the Standard 1-Trailer) moving in the passive lining-up maneuver (see, [9]), where V_{2i-2} plays a role of a tractor whereas V_{2i} a role of a trailer. Therefore, we can use the results presented in [9] to write the dynamics of virtual joint angles between the vehicles V_{2i} and V_{2i-2}:

$$\dot{\beta}_{2i-1}(t) = \frac{-v_{0c}(t)\,\xi_{2i-2}(t)}{L_{2i-1}}\mathrm{s}\beta_{2i-1}(t) + \eta_{2i-1}(t), \quad (28)$$

$$\dot{\beta}_{2i}(t) = \frac{-v_{0c}(t)\,\xi_{2i-1}(t)}{L_{2i}}\mathrm{s}\beta_{2i}(t) + p_{2i}(\beta_{2i-1}(t), t), \quad (29)$$

where $v_{0c}(t) > 0$ for all $t \geq 0$ (see (20)), $\xi_k \triangleq \prod_{j=1}^{k} \mathrm{c}\beta_j$ is positive under condition (c1) and $\xi_k = 1$ if $k < 1$, $\eta_{2i-1}(t) = p_{2i-1}(\beta_{2i-2}(t), t)$, while $p_k(\beta_{k-1}(t), t) = [v_{0c}(t)\,\xi_{k-2}(t)/L_{k-1}]\mathrm{s}\beta_{k-1}(t)$ is a perturbation term such that $p_k(0, t) = 0$. Note that for $i = 1$ the perturbing term $\eta_1(t) \equiv 0$ in (28). For $i > 1$, the term $\eta_{2i-1}(t)$ is negligibly small for $t \in T_2$, upon conclusion (25), and is asymptotically vanishing, thus its effect can be neglected in the (at least initial phase of) stage (S2). Therefore, one can conclude asymptotic (and, at least, initially monotonic) convergence $\beta_{2i-1}(t) \to 0$ as $t \to \infty$. Introducing now a positive definite function $W_\beta \triangleq 1 - \mathrm{c}\beta_{2i}$, we can assess its time derivative as follows:

$$\begin{aligned}
\dot{W}_\beta &\leq -\frac{v_{0c}\,\xi_{2i-1}}{L_{2i}}\mathrm{s}^2\beta_{2i} + \frac{v_{0c}\,\xi_{2i-2}}{L_{2i-1}(t^*_{2i})}\,|\mathrm{s}\beta_{2i}|\,|\mathrm{s}\beta_{2i-1}| \\
&= -(1-\zeta)\frac{v_{0c}\,\xi_{2i-1}}{L_{2i}}\mathrm{s}^2\beta_{2i} \\
&\quad + |\mathrm{s}\beta_{2i}|\,\xi_{2i-2}\left(\frac{v_{0c}\,|\mathrm{s}\beta_{2i-1}|}{L_{2i-1}(t^*_{2i})} - \zeta\frac{v_{0c}\,\mathrm{c}\beta_{2i-1}(t^*_{2i})}{L_{2i}}\,|\mathrm{s}\beta_{2i}|\right),
\end{aligned}$$

where $\zeta \in (0, 1)$ is a majorization constant referenced in (c3). Note that $\xi_{2i-2} > 0$, $\xi_{2i-1} > 0$, and $\mathrm{c}\beta_{2i-1}(t^*_{2i}) > 0$ upon condition (c1). Moreover, in the above worst-case estimation we have used the following two facts inferred from the previous analysis: $\sup_{t\in T_2} |\beta_{2i-1}(t)| = |\beta_{2i-1}(t^*_{2i})|$, and $\forall\, t \in T_2 \;\; L_{2i-1}(t) = L_{2i-1}(t^*_{2i})$. It is evident now, that $\dot{W}_\beta \leq -(1-\zeta)(v_{0c}\,\xi_{2i-1}/L_{2i})\mathrm{s}^2\beta_{2i}$ if only

$$|\mathrm{s}\beta_{2i}| \geq \chi_{2i}\,|\mathrm{s}\beta_{2i-1}|\,, \quad (30)$$

where $\chi_{2i} = L_{2i}/[\zeta L_{2i-1}(t^*_{2i})\mathrm{c}\beta_{2i-1}(t^*_{2i})] \leq 1$ under the first of conditions formulated in (c3). Since $|\mathrm{s}\beta_{2i}(t^*_{2i})| = |\mathrm{s}\beta_{2i-1}(t^*_{2i})|$, by the basic geometric argument (cf. Fig. 1), the inequality (30) is satisfied at the very beginning of stage (S2), and since $\beta_{2i-1}(t) \to 0$ (as argued above) also $\beta_{2i}(t) \to 0$ as $t \to \infty$. Since the convergence of angle $\beta_{2i-1}(t)$ is (at least initially) monotonous, one concludes upon (30) that

$$\sup_{t\geq t^*_{2i}} |\beta_{2i}(t)| = |\beta_{2i}(t^*_{2i})|\,. \quad (31)$$

Now, by recalling (6) and due to (25), one concludes also that $\theta_{2i-1}(t) \to 0$ and $\theta_{2i}(t) \to 0$ as $t \to \infty$, fulfilling the second condition required by (10). Since $\forall\, k\;\boldsymbol{J}_k(0) = \mathrm{diag}\{0, 1\}$, it is clear from (13)-(14) that convergence $\beta_{2i-1}(t) \to 0$ and $\beta_{2i}(t) \to 0$ as $t \to \infty$ implies $(\boldsymbol{u}_{2ic}(t) - \boldsymbol{u}_{2i-2c}(t)) \to \mathbf{0}$ as $t \to \infty$. The latter convergence will happen for consecutive vehicles V_{2i} with $i = 1, 2, \ldots$ subsequently entering the control stage (S2). It leads to the sequence $(\boldsymbol{u}_{2c}(t) - \boldsymbol{u}_{0c}(t)) \to \mathbf{0}$, $(\boldsymbol{u}_{4c}(t) - \boldsymbol{u}_{2c}(t)) \to \mathbf{0}, \ldots$ as $t \to \infty$, which implies $(\boldsymbol{u}_{2ic}(t) - \boldsymbol{u}_{0c}(t)) \to \mathbf{0}$ as $t \to \infty$. Since $\boldsymbol{u}_0(t) := \boldsymbol{u}_{0c}(t)$ and $\boldsymbol{u}_{2i}(t) := \boldsymbol{u}_{2ic}(t)$ (by Proposition 1), the third condition imposed by (10) will be fulfilled.

Remark 5: A rationale behind assuming that condition (c2) when satisfied at $t = t^*_{2i}$ is also preserved for any finite $t \in T_2$ comes from the following reasoning. Satisfaction of (c2) at $t = t^*_{2i}$ corresponds to satisfaction of inequality (23) at $t = t^*_{2i}$. At the beginning of stage (S2) holds $s_{2i}(t^*_{2i}) = 1$ (*switching transition*), thus upon (26) one can state that $\dot{L}_{2i-1}(t^*_{2i}) = 0$. If the joint angles $\beta_{2i-1}(t)$ and $\beta_{2i}(t)$ for $t \geq t^*_{2i}$ converge to zero in such a way that $\sup_{t\in T_2} |\beta_{2i-1}(t)| = |\beta_{2i-1}(t^*_{2i})|$ and $\sup_{t\in T_2} |\beta_{2i}(t)| = |\beta_{2i}(t^*_{2i})|$, the inequality (23) will be satisfied also for all finite $t > t^*_{2i}$.

Evolution of the y-positional error in stage (S2) results from the following perturbed differential equation

$$\dot{e}_{2i}(t) = -\frac{v_{2i}(t)}{L_{2i}} e_{2i}(t) + g_{2i}(t) = f_{2i}(e_{2i}, t) + g_{2i}(t) \quad (32)$$

which can be directly derived by differentiation of (9), using kinematics (1), and utilizing a basic geometrical relation $e_{2i} = L_{2i}\mathrm{s}\theta_{2i} + L_{2i-1}\mathrm{s}\theta_{2i-1}$ (cf. Fig. 1). The perturbing term $g_{2i}(t) = v_{2i-2c}(t)[\frac{L_{2i-1}(t)}{L_{2i}}\mathrm{c}\beta_{2i}(t)\mathrm{c}\beta_{2i-1}(t)\mathrm{s}\theta_{2i-1}(t) + \mathrm{s}\theta_{2i-2}(t)]$ satisfies $|g_{2i}(t)| \leq \sigma_{2i}(t)$ for all $t \in T_2$, where the function $\sigma_{2i}(t)$ has been introduced in (18). According to (32), it is clear that the equilibrium $e_{2i} = 0$ of the unperturbed dynamics $\dot{e}_{2i}(t) = f_{2i}(e_{2i}, t)$ is globally exponentially stable, which can be easily checked by using a Lyapunov function $W_e \triangleq e_{2i}^2/2$ and recalling the result (20). In view of the above, and by referring to the Lemma 9.4 formulated in [6], one can conclude that the solution of (32) satisfies

$$\forall\, t \in T_2 \;\; |e_{2i}(t)| \leq |e_{2i}(t_{2i}^*)| \exp(-\alpha(t - t_{2i}^*)) + \mu_{2i}(t) \quad (33)$$

with $\mu_{2i}(t) = \int_{t_{2i}^*}^{t} \sigma_{2i}(\tau) \exp(-\alpha(t-\tau)) d\tau$, and $\alpha = \frac{v_m}{L_{2i}}$. Moreover, since $\sigma_{2i}(\tau) \to 0$ as $t \to \infty$ (because $\theta_{2i-1}(t) \to 0$ and $\theta_{2i-2}(t) \to 0$ as $t \to \infty$, as shown before), by recalling the Lemma 9.6 formulated in [6], one concludes that $e_{2i}(t) \to 0$ as $t \to \infty$ which fulfills the first condition imposed by (10).

Next, let us analyze satisfaction of guarantee (g3). According to (33), one can write (see [6], pp. 352)

$$\sup_{t \in T_2} |e_{2i}(t)| \leq \max\left\{ |e_{2i}(t_{2i}^*)| \,;\, \frac{\bar{\sigma}_{2i}}{\alpha} \right\} \leq |e_{2i}(t_{2i}^*)| + \frac{\bar{\sigma}_{2i}}{\alpha}$$

where $\bar{\sigma}_{2i} = \sup_{t \in T_2} \sigma_{2i}(t)$ (see (18)). Furthermore, upon definition (9), it can be checked that $e_{2i}(t) = y_0 - y_{2i}(t) - \sum_{k=1}^{i-1} e_{2k}(t)$. Moreover, for the sufficiently small constant ϵ_{2i} introduced in (c5), the errors $|e_{2k}(t)|$ for $k < i$ and $t \geq t_{2i}^*$ converge monotonously implying $\sup_{t \in T_2} \sum_{k=1}^{i-1} |e_{2k}(t)| = \sum_{k=1}^{i-1} |e_{2k}(t_{2i}^*)|$. As a consequence, one can assess:

$$\begin{aligned} \sup_{t \in T_2} |y_0 - y_{2i}(t)| &\leq \sup_{t \in T_2} |e_{2i}(t)| + \sup_{t \in T_2} \sum_{k=1}^{i-1} |e_{2k}(t)| \\ &\leq |e_{2i}(t_{2i}^*)| + \frac{\bar{\sigma}_{2i}}{\alpha} + \epsilon_{2i} \\ &= \left| y_0 - y_{2i}(t_{2i}^*) - \sum_{k=1}^{i-1} e_{2k}(t_{2i}^*) \right| + \frac{\bar{\sigma}_{2i}}{\alpha} + \epsilon_{2i} \\ &\leq |y_0 - y_{2i}(0)| + \frac{\bar{\sigma}_{2i}}{\alpha} + 2\epsilon_{2i}, \end{aligned} \quad (34)$$

where we utilized the fact that $y_{2i}(t_{2i}^*) = y_{2i}(0)$ (because $\forall\, i \;\dot{y}_{2i}(t) \equiv 0$ within (S1)). The result (34), together with assumption A2, indicate that the absolute y-position of the vehicle V_{2i} cannot increase by more than $2\epsilon_{2i} + \bar{\sigma}_{2i}/\alpha$ with respect to its absolute initial y-position. By referring to assumption A1, the worst case scenario imposes the safety condition $2\epsilon_{2i} + \bar{\sigma}_{2i}/\alpha < \varepsilon_{2i}$, corresponding to (17), which implies satisfaction of guarantee (g3).

Finally, let us justify the forms of conditions (c3) and (c4) in the context of guarantee (g4). When the vehicle V_{2i} enters the control stage (S2), it should preserve safe distances $d_{2i,2i-2}$ and $d_{2i+2,2i}$ between the two neighboring vehicles: V_{2i-2} and V_{2i+2}, respectively. From basic geometry, the distance $d_{2i,2i-2}$ results from the following formula: $d_{2i,2i-2}(t) = [L_{2i}^2 + 2L_{2i}L_{2i-1}(t)\mathrm{c}\beta_{2i}(t) + L_{2i-1}^2(t)]^{1/2}$. According to the results (27) and (31), one observes that a lower bound of the distance $d_{2i,2i-2}(t)$ for $t \in T_2$ satisfies $\inf_{t \in T_2} d_{2i,2i-2}(t) = d_{2i,2i-2}(t_{2i}^*)$, where

$$d_{2i,2i-2}^2(t_{2i}^*) = L_{2i}^2 + 2L_{2i}L_{2i-1}(t_{2i}^*)\mathrm{c}\beta_{2i}(t_{2i}^*) + L_{2i-1}^2(t_{2i}^*),$$

while (since $\beta_{2i}(t \to \infty) \to 0$, as shown before)

$$d_{2i,2i-2}(\infty) = L_{2i} + L_{2i-1}(t_{2i}^*). \quad (35)$$

Now, by imposing $d_{2i,2i-2}^2(t_{2i}^*) \geq d_{\min}^2$ required by (g4) leads to the second part of condition (c3). A lower bound of the distance $d_{2i+2,2i}(t)$ for $t \in T_2$ can be (conservatively) assessed taking into account only its component $d_{2i+2,2i}^x$ along the x^G axis. It is clear that $d_{2i+2,2i-2}^x(t) = x_{2i-2}(t) - x_{2i+2}(t)$ and $\dot{d}_{2i+2,2i-2}^x(t) = v_{2i-2}(t)\mathrm{c}\theta_{2i-2}(t) - v_{2i+2}(t)\mathrm{c}\theta_{2i+2}(t) \approx (v_{2i-2}(t) - v_{2i+2}(t)) \geq 0$ for $t \in T_2$ (the latter approximation comes from a fact that the vehicle V_{2i+2} is still in control stage (S1), while the vehicle V_{2i-2} has almost finished its MPF maneuver). Therefore, we can write $d_{2i+2,2i-2}^x(t_{2i}^*) \leq d_{2i+2,2i-2}^x(\infty)$ and, as a consequence, $d_{2i+2,2i}^x(t_{2i}^*) + d_{2i,2i-2}^x(t_{2i}^*) \leq d_{2i+2,2i}^x(\infty) + d_{2i,2i-2}^x(\infty)$. From the latter inequality we have

$$d_{2i+2,2i}^x(\infty) \geq d_{2i+2,2i}^x(t_{2i}^*) + d_{2i,2i-2}^x(t_{2i}^*) - d_{2i,2i-2}^x(\infty).$$

Since the lower bound of the distance between the vehicles V_{2i+2} and V_{2i} satisfies $\inf_{t \in T_2} d_{2i+2,2i}^x(t) \geq d_{2i+2,2i}^x(\infty)$, we can impose a conservative condition motivated by (g4)

$$d_{2i+2,2i}^x(t_{2i}^*) + d_{2i,2i-2}^x(t_{2i}^*) - d_{2i,2i-2}^x(\infty) \geq d_{\min} \quad (36)$$

to guarantee no collision between vehicles V_{2i+2} and V_{2i} within the control stage (S2). From the basic geometry (cf. Fig. 1) and since $\theta_{2i}(\infty) = 0$ (as shown before) we know that $d_{2i,2i-2}(\infty) \equiv d_{2i,2i-2}^x(\infty)$, and by recalling that $\theta_{2i+2}(t_{2i}^*) = \theta_{2i}(t_{2i}^*) = 0$ we can write: $d_{2i+2,2i}^x(t_{2i}^*) = L_{2i+2} + L_{2i+1}(t_{2i}^*)\mathrm{c}\beta_{2i+2}(t_{2i}^*)$, $d_{2i,2i-2}^x(t_{2i}^*) = L_{2i} + L_{2i-1}(t_{2i}^*)\mathrm{c}\beta_{2i}(t_{2i}^*)$, and (see (35)) $d_{2i,2i-2}^x(\infty) = L_{2i} + L_{2i-1}(t_{2i}^*)$. Substituting all these formulas into inequality (36), and resolving it with respect to $L_{2i+1}(t_{2i}^*)$, gives condition (c4). Summarizing, satisfaction of (c3)-(c4) entails satisfaction of (g4).

V. Simulation results and comments

We present exemplary simulation results obtained for a set of five car-like vehicles moving along a road of width $D = 5.5\,\mathrm{m}$ and satisfying assumptions A1-A4 (for $\Delta_{2i} = 58°$ and $l_{2i} = 2.65\,\mathrm{m}$). An initial configuration of the ordered vehicles has been illustrated on the left side in the middle plot in Fig. 3. The following parameters have been selected for simulations: $L_{2i} = 2.5\,\mathrm{m}$ for all $i \in \{1, 2, 3, 4\}$, $d_{\min} = 4.5\,\mathrm{m}$, $v_m = 10\,\mathrm{m/s}$, $v_M = 1.5 v_m = 15\,\mathrm{m/s}$, $\epsilon_{2i} = 0.2\,\mathrm{m}$, for all $i \in \{1, 2, 3, 4\}$, $t_0^* = 1\,\mathrm{s}$, and $T_\alpha = 2\,\mathrm{s}$. Transitions from the control stage (S1) to stage (S2) for particular vehicles V_{2i} were forced by using the transition operator (15) with the scaling factor $T_s = 2\,\mathrm{s}$. As a consequence, the subsequent

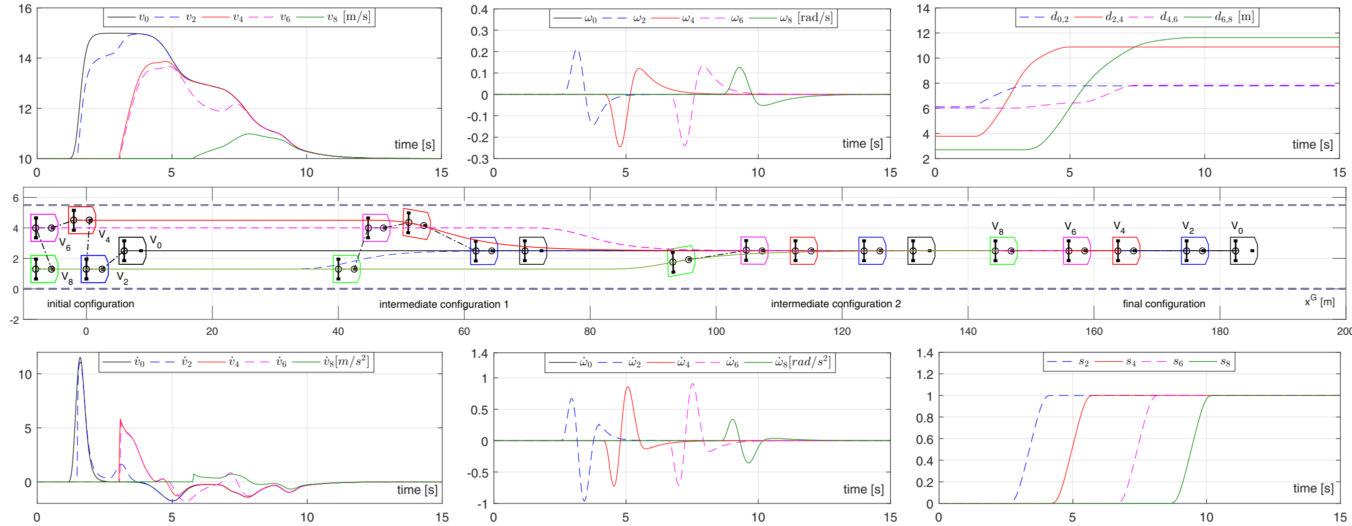

Fig. 3. Simulation results of the MPF maneuvers for the case of the five-vehicle set $\mathcal{S} = \{V_0, V_2, V_4, V_6, V_8\}$; four selected configurations of the multi-vehicle system have been illustrated on the middle plot (thin dash-dot lines illustrate the links forming the virtual N-trailer); accelerations presented on the bottom plots have been numerically approximated (see the animation at https://goo.gl/Gsa1L6)

transition processes for particular vehicles were smooth (see the obtained velocity profiles in Fig. 3), and every transition duration was equal exactly T_s seconds.

Analyzing the results presented in Fig. 3, one may observe that the control process starts at $t = t_0^* = 1\,$s and practically finishes in less than $15\,$s. The platoon has been safely formed keeping the vehicles within the road width D, keeping the inter-vehicle distances non-decreasing, and preserving the order of vehicles prescribed at the beginning of maneuvers. The results reveal conservativeness of condition (17) – in the considered case the y-positions of all the merging vehicles were converging monotonically within the whole control stage (S2) yielding $\sup_{t \in T_2} |y_0 - y_{2i}(t)| = |y_0 - y_{2i}(0)|$ (cf. (34)). It is worth stressing that longitudinal and angular velocities (satisfying all the imposed constraints) have been smoothed due to the usage of transition operators s_{2i} (cf. the right-bottom plot in Fig. 3) with scaling factor $T_s \gg 0$. This fact reveals that the non-instantaneous transitions from control stage (S1) to stage (S2) may not only preserve the expected safety guarantees of the maneuvers, but also helps lowering the maximal absolute accelerations of the vehicles, which seems to be an important practical benefit (for numerically approximated acceleration profiles, see the left-bottom and the middle-bottom plots in Fig. 3).

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