

Lyapunov Global Stability for a Reactive Mobile Robot Navigation in Presence of Obstacles

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Abstract—This paper deals with the navigation of a mobile robot in unknown environment. The robot has to reach a final target while avoiding obstacles. It is proposed to break the task complexity by dividing it into a set of basic tasks: Attraction to a target and obstacle avoidance. Each basic task is accomplished through the corresponding elementary controller. The activation of one controller for another is done according to the priority task. To ensure the overall stability of the control system, especially at the switch moments, properties of hybrid systems are used. Hybrid systems allow switching between continuous states in presence of discrete events. In this paper, it is proposed to act on the gain of the proposed control law. The aim is to ensure the convergence of a common Lyapunov function to all the controllers. This ensures the stability of the overall control. Simulation results confirm the theoretical study.

I. INTRODUCTION

The control of a mobile robot navigating in a cluttered environment is a fundamental problem and is receiving much attention in the robotics community. The purpose is mainly to ensure to the mobile robot a suitable and a safe navigation (avoiding a risk of collision, respecting its structural constraints, etc..)

Some of the literature considers that the robot control is entirely based on the methods of path planning while involving the total or partial knowledge of its environment: Voronoi diagrams and visibility graphs [1] or Artificial potential fields functions containing all the information on the target [2] and the robot environment are among these methods. Another community is interested by the ability of the robot to achieve the control laws according to its constraints (structural constraints, jerk-control, etc.). Even if cognitive methods of path planning and replanning [3], [4], can also be found here, more reactive methods (based on sensors information rather than a prior knowledge of the environment) are more common [5], [6] or [7]. The proposed work falls into the latter approach.

To ensure the robot's ability to accomplish a reactive task, it is proposed to explore behavioral control architectures originally proposed by Brooks [8]. This kind of architecture of control breaks the complexity of the overall task by dividing it into several basic tasks. Each basic task is accomplished with its corresponding controllers. There are two major principles of coordinating them: the action selection [8] and merging actions [9]. In the first, only one controller selected from the basic controllers is applied to the robot at every sample time. In the second case, the control applied

to the robot is a result of merging all or a part of available controllers in the control architecture.

We note that the action selection is more interesting. Indeed, one controller is applied to the mobile robot at a given time. It is then easier to examine individual stability of each controller. However, random switch from one controller to another (avoiding obstacles, follow a trajectory, reaching a target, etc.) may cause instability of the global control law, even if each individual controller is stable [10].

Stability proof of this kind of control architecture has been little explored in the literature: in [5], a merging action node is introduced to the control automaton in order to smoothly switch between the two controllers. The advantage of studying each controller alone is then lost, since we have also to study the merging action node. Controlling a mobile robot to follow a trajectory in presence of obstacles, based on the theorem of multiple Lyapunov functions [10] was established in [11]: A third secondary controller was then introduced to satisfy this theorem. However, this control architecture is not suitable for any cluttered environment.

Finding a common Lyapunov function to the basic systems forming a hybrid system is not a simple task [12]. In this paper, we propose to deal with this problem by ensuring overall stability of our control architecture with a single Lyapunov function. Here we are interested by a mobile robot reaching a target while avoiding obstacles: this task is then divided into two basic tasks: attraction to a target and obstacle avoidance.

The rest of the paper is organized as follows: in next section, the basic controllers and the proposed control law are introduced. The proposed control architecture is exposed in Section III. Simulation results are given in IV. Finally, we conclude and give some prospects in Section V.

II. ROBOT MODEL AND TASKS TO ACHIEVE

Before introducing attraction to the target controller, obstacle avoidance and the proposed control law, we recall that the kinematic model of the used unicycle mobile robot used is expressed by the well-known equations:

$$\begin{aligned}\dot{x} &= v\cos(\theta) \\ \dot{y} &= v\sin(\theta) \\ \dot{\theta} &= \omega\end{aligned}\tag{1}$$

With

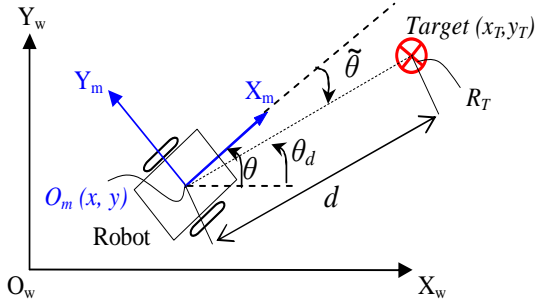


Fig. 1. Controller for attraction to target.

- (x, y) are the world coordinates of the robot axle center O_m (cf. Figure 1).
- θ is the world robot orientation.
- v and ω are respectively linear and angular velocities.

A. Attraction to target controller

The robot has to reach a given target of radius R_T and coordinates center (x_T, y_T) (cf. Figure 1).

Position errors are defined as

$$\begin{aligned} e_x &= x_T - x = d \cos(\tilde{\theta}) \\ e_y &= y_T - y = d \sin(\tilde{\theta}) \end{aligned} \quad (2)$$

d is the distance of the robot to the target and can then be expressed as

$$d = \sqrt{e_x^2 + e_y^2} \quad (3)$$

$\tilde{\theta}$ is the orientation error, such that $\tilde{\theta} \in]-\pi, \pi]$ is

$$\tilde{\theta} = \tan^{-1}\left(\frac{y_c - y}{x_c - x}\right) - \theta \quad (4)$$

Its derivative $\dot{\tilde{\theta}}$ is then

$$\dot{\tilde{\theta}} = \left(\frac{\dot{e}_y}{e_x}\right) / \left(1 + \left(\frac{e_y}{e_x}\right)^2\right) - \omega \quad (5)$$

After computation using the kinematic model (cf. Equation 1) and equations in (2) we obtain

$$\dot{\tilde{\theta}} = \omega_r - \omega \quad (6)$$

Where

$$\omega_r = v \frac{\sin(\tilde{\theta})}{d}$$

B. Obstacle avoidance controller

The objective of this controller is to control the robot to avoid obstacles that hinder its attraction to the target. To focus on the proposed control architecture, this controller is briefly described. The theoretical details are available in [13].

This controller is based on the limit cycle methods [14], [15]. The differential equations representing the desired trajectory of the robot are given by the following system

$$\begin{aligned} \dot{x}_r &= ay_r + x_r(R_c^2 - x_r^2 - y_r^2) \\ \dot{y}_r &= -ax_r + y_r(R_c^2 - x_r^2 - y_r^2) \end{aligned} \quad (7)$$

With $a = \pm 1$ according to the optimal direction of avoidance (clockwise or counterclockwise direction). (x_r, y_r) are

the relative robot coordinates with respect to the obstacle. The latter is characterized by a circle of radius $R_{cl} = R_o + R_r + \epsilon$ where: R_o is the obstacle radius, R_r is the robot radius and ϵ is a safety margin (cf. Figure 2).

The algorithm for obstacle avoidance is summarized in the following

- The nearest hindering obstacle is detected.
- The direction of avoidance is chosen according to the sensor information.
- The robot avoids the obstacle while following a limit cycle which has a radius $R_c = R_{cl} - \xi$ (attraction phase).
- The robot avoids the obstacle while following a limit cycle which has a radius $R_c = R_{cl} + \xi$ (repulsive phase) (cf. Figure 2). Where ξ is a small value and $(\xi \ll \epsilon)$.

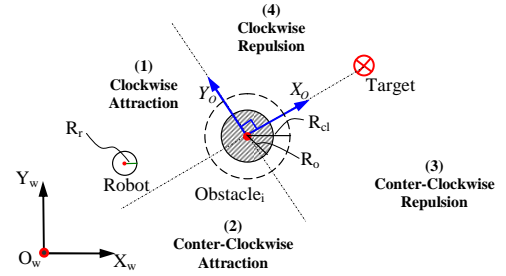


Fig. 2. Obstacle avoidance controller.

C. The proposed control law

It is interesting to notice that only one control law is applied to the robot even if its architecture of control contains two different controllers (attraction to the target and obstacle avoidance). Only the set points change according to the applied controller.

The proposed control law is expressed as follows:

$$\begin{aligned} v &= v_{max} e^{-\frac{1}{d} \cos(\tilde{\theta})} \quad (a) \\ \omega &= \omega_r + k_1 \dot{\tilde{\theta}} \quad (b) \end{aligned} \quad (8)$$

where

- v_{max} is the maximum linear velocity.
- k_1 is a constant such that $k_1 > 0$.
- d is the distance robot-target (cf. Equation 3). The robot reaches the target when $0 < d \leq R_T$ (cf. Section II-A).

To study the stability of the proposed control law, consider the Lyapunov function

$$V = \frac{1}{2} \tilde{\theta}^2$$

The control law is asymptotically stable if $\dot{V} < 0$.

$$\dot{V} = \tilde{\theta} \dot{\tilde{\theta}}$$

By replacing (6) in (8.b), we get

$$\dot{\tilde{\theta}} = -k_1 \tilde{\theta} \quad (9)$$

and \dot{V} becomes

$$\dot{V} = -k_1 \tilde{\theta}^2 < 0 \quad (10)$$

and \dot{V} becomes for every $\tilde{\theta} \neq 0$.

The controller is then asymptotically stable.

Once each basic task and the control law are defined, the proposed architecture of control which coordinates them is given in next section.

III. THE PROPOSED ARCHITECTURE OF CONTROL

Even if each controller is individually stable, it is important to constrain switch between them to avoid instability of the overall system, see [10]. Here, it is proposed to generalize the Lyapunov function previously defined (cf. Section II-C) for the overall control system. Indeed, it was proved (cf. Section II-C) that this function is strictly decreasing. However, the problem arises (as for all hybrid systems) at switching moments where the set point is discontinuous. This means that there is an unavoidable jump of the error $\tilde{\theta}$ at these moments. This naturally leads to jumps in the Lyapunov function after the switch and this jump may lead to increasing it.

Hence, It is proposed to adjust the gain k_1 of the control law (cf. Equation 8) at the switch moments so that even if the value of the Lyapunov function increases during the switch, it returns to its value before switch $V(t_{bs})$ in a finite time T_{max} . (t_{bs} is the moment just before switch).

In addition, the robot should not navigate more than a distance d_{max} when ($V(t) > V(t_{bs})$) in order to insure stability criterion as soon as possible. Also, when the robot performs the obstacle avoidance task, it is necessary that ($d_{max} < \epsilon$) (cf. Section II-B) to avoid collision with the obstacle. Notice that ϵ is the minimal distance separating the robot from the obstacle once this one is detected (cf. Section II-B).

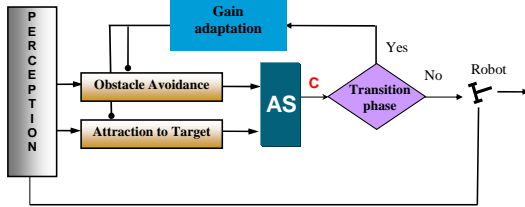


Fig. 3. The proposed architecture of control.

A. Adapting the control law gain

The adjustment of the gain k_1 (cf. Equation 8) is triggered if one of the following events occurs

- The control of the robot switches from one controller to another.
- Obstacle avoidance controller switches from an obstacle to another.
- Obstacle avoidance moves from attraction phase to repulsive phase (cf. Figure 2).

To insure that V decreases in a finite time T_{max} that we can impose, we have to get

$$V(t_s + T_{max}) \leq V(t_{bs}) \quad (11)$$

Where t_s is the switch moment.

The resolution of the differential equation (9) gives the orientation error with respect to time $\tilde{\theta}$

$$\tilde{\theta}(t) = \tilde{\theta}(t_s) e^{-k_1(t-t_s)} \quad (12)$$

Equation (12) allows to easily deduce the Lyapunov function :

$$\begin{aligned} \tilde{\theta}^2(t) &= \tilde{\theta}^2(t_s) e^{-2k_1(t-t_s)} & (a) \\ V(t) &= \frac{\tilde{\theta}^2(t_s)}{2} e^{-2k_1(t-t_s)} & (b) \\ V(t) &= V(t_s) e^{-2k_1(t-t_s)} & (c) \end{aligned} \quad (13)$$

Thus, k_1 is expressed as

$$k_1 = \frac{\ln(V(t)/V(t_s))}{-2(t-t_s)} \quad (14)$$

Note that k_1 is always positive. Indeed, $V(t) \leq V(t_s)$ (cf. Equation 13) and then $\ln(\frac{V(t)}{V(t_s)}) \leq 0$.

The value of k_1 allowing to reach $V(t_{bs})$ in a finite time T_{max} is

$$k_1 = \frac{\ln(V(t_{bs})/V(t_s))}{-2T_{max}} \quad (15)$$

Note that the restriction on T_{max} is necessary especially in the case of obstacle avoidance. Indeed, the stability criterion of hybrid systems (cf. Equation 11) must be satisfied in minimal time. Moreover, the distance achieved during T_{max} has to be ($d_{max} \leq \epsilon$) (cf. Section II-B) to avoid collision with the obstacle. It is easy to see that the minimum necessary time to achieve this distance is

$$t_{min} = \frac{\epsilon}{v_{max}}$$

corresponding to a straight robot navigation to the obstacle center with its maximum linear velocity. (15) becomes then

$$k_1 = \frac{\ln(V(t_{bs})/V(t_s))}{-2t_{min}} \quad (16)$$

Note that k_1 is not defined if $V(t_{bs}) = 0$. The notion of weak stability [16] allows to define a threshold V_{min} such that if $V(t) < V_{min}$, then the system is (weakly) stable without comparing $V(t)$ to $V(t_{bs})$. It means that

$$k_1 = \frac{\ln(V_{min}/V(t_s))}{-2t_{min}} \quad (17)$$

Thus, k_1 is recalculated in this way and replaced in (8.b).

We can then summarize the proposed control architecture as in figure (Fig. 3).

B. The mechanism of the architecture of control

The block *AS* (for *Action Selection*) selects the suitable controller to apply to the robot according to the environment: if no obstacle is detected, Attraction to the target task is accomplished. If there is a discrete event (switching from one controller to another, transition from attraction to repulsive phase, etc.), the block *transition phase* prevents the control from affecting the robot's actuators, until the block *adaptation gain* recalculates the gain k_1 as previously highlighted.

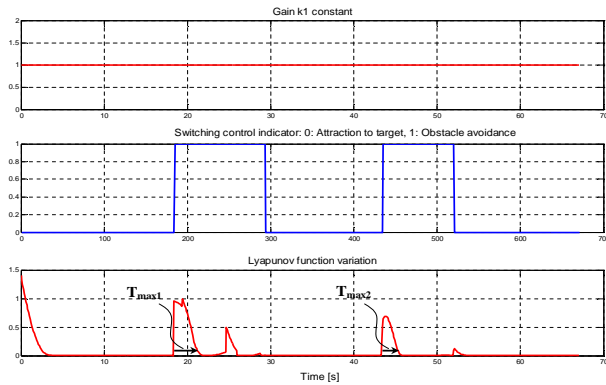


Fig. 4. Variation of the Lyapunov function keeping k_1 constant.

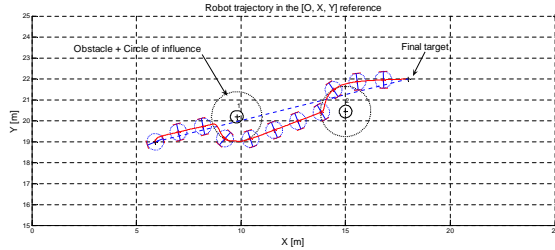


Fig. 5. Robot trajectory in presence of obstacles.

IV. SIMULATION RESULTS

To estimate the relevance of the proposed control architecture, it is proposed to simulate a mobile robot navigation to reach a target in presence of obstacles. Simulation is made twice. In the first case, the used control law has a constant gain during all the navigation ($k_1 = 1$) (there is no gain adjustment in the switch moments). Switching control indicating the active controller can be seen in figure (Fig. 4).

In the second case, the proposed control architecture is implemented on the robot. In the two cases, the robot reaches its target while avoiding obstacles. However, by comparing T_{max1}, T_{max2} which are convergence times for obstacle avoidance controller in figures (Fig. 4) and (Fig. 6), it is noticed that the Lyapunov function of the proposed architecture of control converges faster than the architecture with a constant gain. Evolution of the gain k_1 is given in the same figure (Fig. 6). Note that attraction to the target controller converges fastly in the two cases even if in the proposed architecture, we can see that it is slightly faster.

V. CONCLUSION

A control architecture based on hybrid systems has been proposed. With these systems, it is possible to divide the control architecture into a set of elementary controllers to examine each controller separately. Even if each individual controller is stable, global stability is not necessarily guaranteed. In this paper, the overall stability was established thanks to a single Lyapunov function. The proposed idea is to adjust the gain of the control law in order to accelerate convergence of the Common Lyapunov Function CLF after each switch. The simulation results have confirmed the theoretical study. In future works, it is proposed to introduce the gain k_1 as a

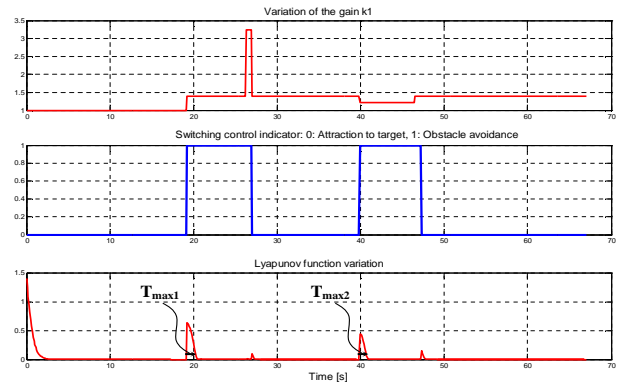


Fig. 6. Variation of the Lyapunov function and the gain k_1 with the proposed architecture of control.

dynamical gain. Thus, once the lyapunov function converges, it returns to its nominal value without disturbing the control.

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